Eddy Currents and Magnetic Calibrations in LDX using a “Copper Plasma”

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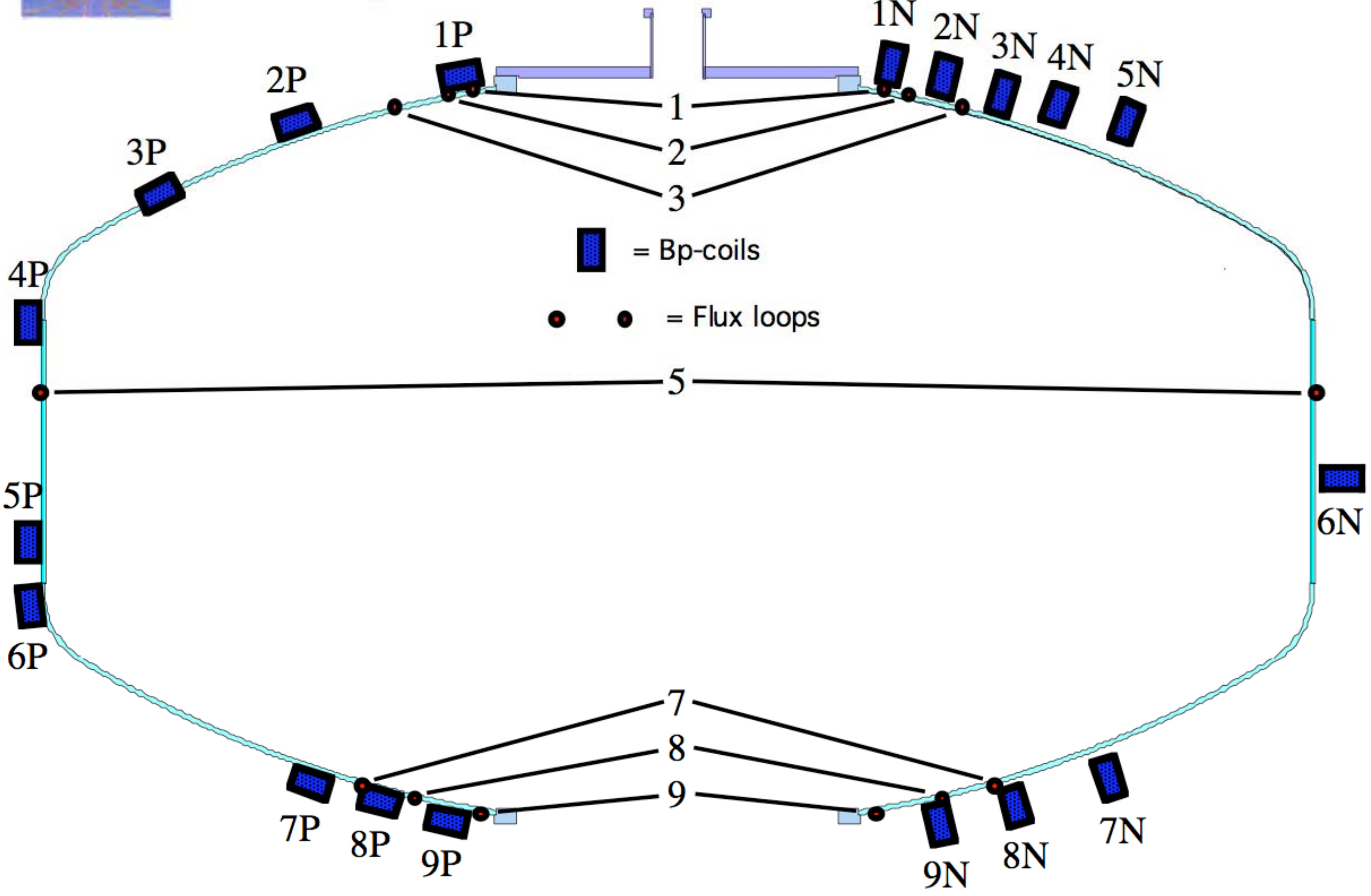
“Copper Plasma” Overview

- LDX Magnetics
- Goals
  - Calibrate magnetic diagnostics positions and gains
  - Find eddy current decay times
- Copper Plasma Operation
- Theory
- Analysis
- Results

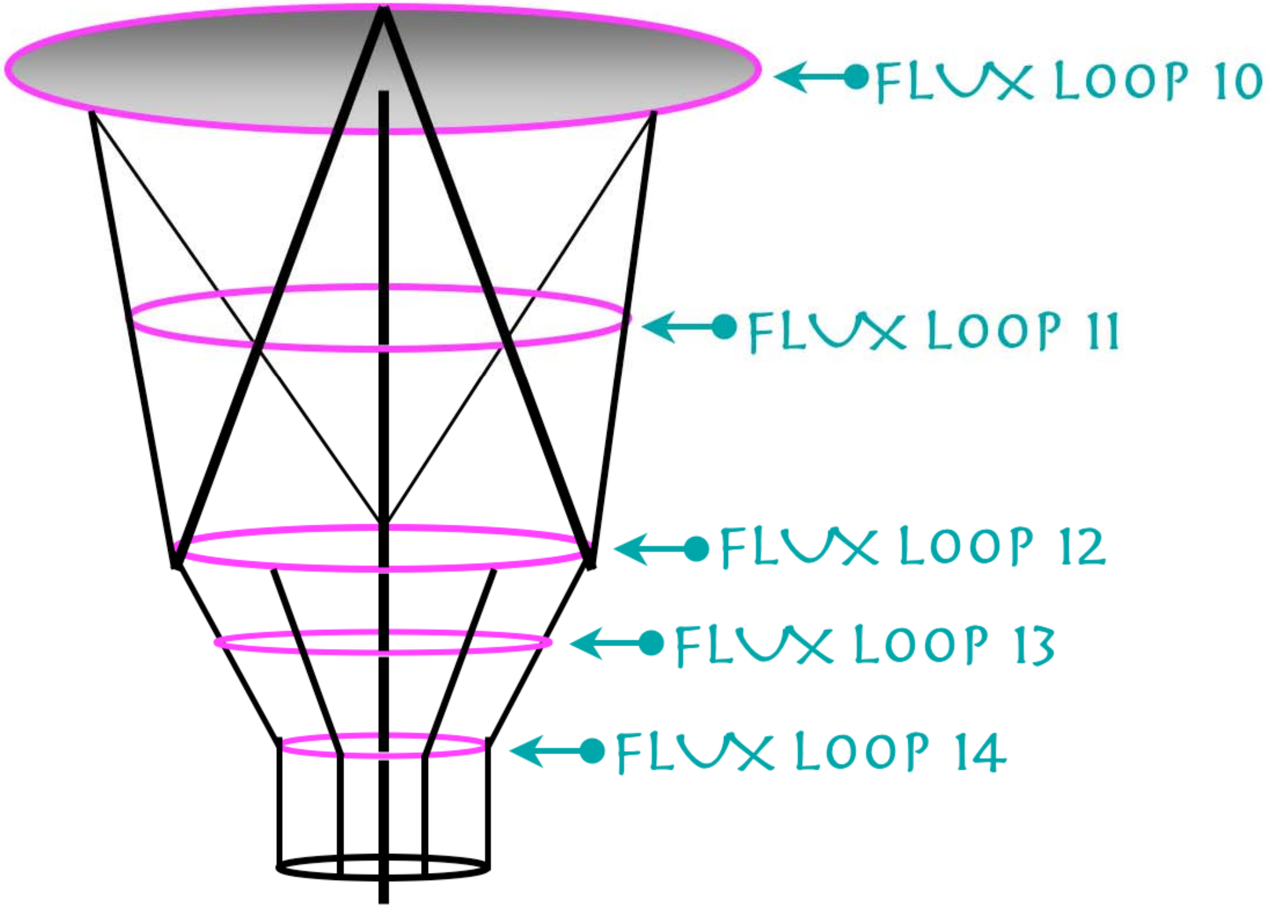
LDX Magnetics

- 18 external magnetic field sensors
  - 9 parallel
  - 9 normal
  - ***n*** ≈ 1000 turns, ***a*** ≈ 50 cm^2, 15 cm length
- 9 external flux loops
- 5 internal flux loops
- Sensors are time integrated using RC circuits

Sensor positions:



Upper Catcher with Interal Flux Loops




1
2
3
4
5
6
SHTAKE

$$\vec{A}(r,z,R,Z,I)=\mu_0 IR\frac{(2-k^2)K(k^2)-2E(k^2)}{\pi k^2\sqrt{(r+R)^2+(z-Z)^2}}\hat{\varphi}$$

$$k^2\equiv\frac{4rR}{(r+R)^2+(z-Z)^2}$$

$$B_z=\frac{1}{r}\frac{\partial}{\partial r}rA_\varphi \;\; ; \;\; B_r=-\frac{\partial}{\partial z}A_\varphi$$

$$\Phi=\int\vec{B}\cdot d\vec{a}=\oint\vec{A}\cdot d\vec{\ell}=2\pi rA$$

$$\Phi=\int\vec{B}\cdot d\vec{a}\approx\vec{B}\cdot n\vec{a}=na(B_z\cos\theta+B_r\sin\theta)$$

$$V=\frac{1}{RC}\int-\frac{d\Phi}{dt}dt=-\frac{\Phi}{RC}$$

Goal: Calibrate Magnetics

- Positions of sensors ***r, z, θ*** have been measured with rulers and laser level
    - Accuracy ~ 3mm / 1º
- Gains ***g*** of integrator boards have been measured electronically
- Would be better to measure positions and total gains **magnetically**

Goal: Eddy Currents

- Magnetic levitation vertically unstable
  - Requires active feedback control
  - Fluctuations in L-coil, F-coil and plasma current magnitude
  - Fluctuations in F-coil and plasma position
- Changing currents induce eddy currents in vessel
  - Eddy currents are picked up by magnetic diagnostics
  - Must be measured so they can be properly included in magnetic reconstruction
  - L- and F-coil eddy currents measured in vacuum shots
    - L-coil only shots – know position, know current
    - Use L-coil to induce vertical jog in F-coil, measure position with lasers and calculate current from force balance
- **How to estimate eddy currents due to fluctuating plasma currents?**
  - Changes in plasma current magnitude and position **NOT** known

Solution: Build “Copper Plasmas”

- 2 coils of 12 AWG copper wire
  - Approximately same radius as plasma
  - ***R*** = 76 cm coil - 28 turns, 91 cm coil - 25 turns
  - Built on plywood forms, forms disassembled and reassembled in vessel, coils squeezed through port
- Operated in 3 vertical positions
  - From midplane, ***Z*** = 0, ±5 cm
- L-coil control system used to impose trapezoidal current pulses
  - 4s flattop @ 60A
  - Rise and fall times: 0.5s, 1s, 2s
- Operated with + and - polarity by switching coil leads
- 2 ***R*** x 3 ***Z*** x 3 times x 2 polarities x 31 sensors

Photo of copper plasma

All copper plasma parameters for 2 coils in 3 vertical positions given by :

$$\overrightarrow{X}_c \equiv \{R_1, R_2, Z_1, Z_2, Z_3, \delta Z, \Theta_1, \Theta_2, \Theta_3, \delta\Theta, \rho_1, \rho_2, \rho_3, \delta\rho, I\}$$

For copper plasma shot i, sensor j :

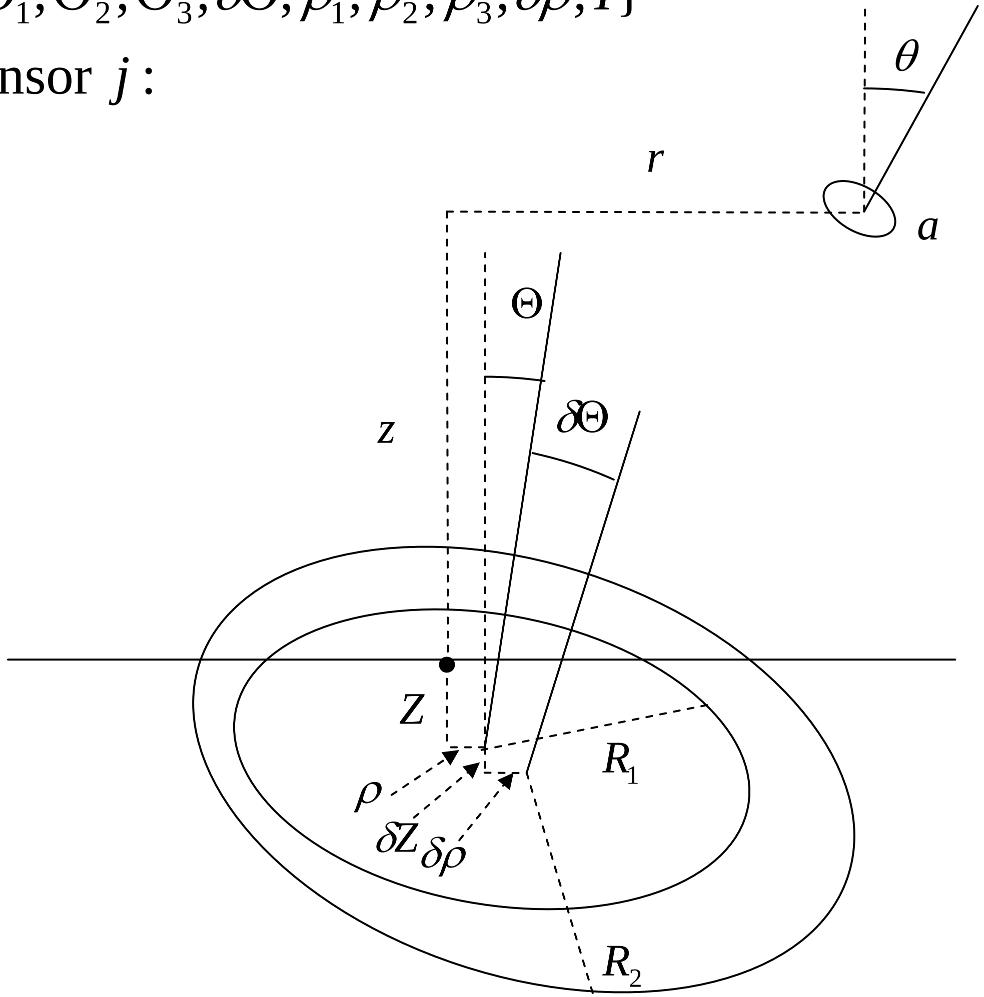
$$V_{ij} = V_{ij}(\vec{x}_{ci}, \vec{x}_{sj})$$

Copper Plasma parameters :

$$\vec{x}_{ci} \equiv \{R_i, Z_i, \Theta_i, \rho_i, I_i\}$$

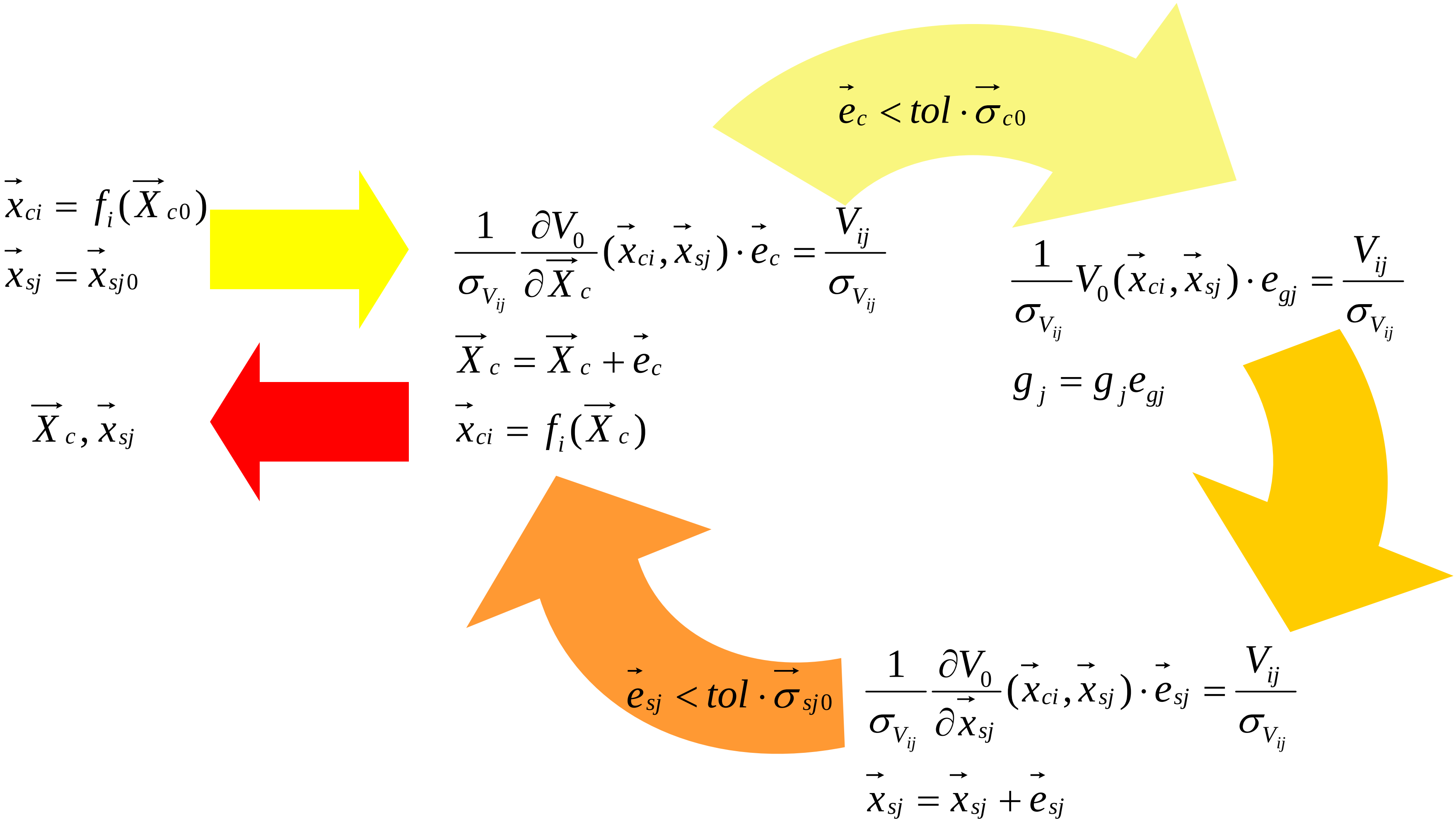
Sensor parameters :

$$\vec{x}_{sj} \equiv \{r_j, z_j, \theta_j, g_j\}$$



Theory: Calibrate Magnetics

- By using copper plasma with 2 different radii $\boldsymbol{R}$ at 3 different heights $\boldsymbol{Z}$, can use least squares fit to find sensor calibration
    - 6 equations to find 3 or 4 unknowns
- Copper plasma parameters also not known exactly, can use least squares to find them
    - 2 $\boldsymbol{R}$ x 3 $\boldsymbol{Z}$ x 31 sensors = 186 equations to find 15 unknowns
- $\boldsymbol{V}(\boldsymbol{x_c},\boldsymbol{x_s})$ is non-linear
    - Linearize using measured parameters
    - Solve linear least squares
    - Adjust parameters and repeat
    - Switch between solving for $\boldsymbol{x_c}$ and $\boldsymbol{x_s}$



$$\vec{x}_{ci} = f_i(\vec{X}_{c0})$$

$$\vec{x}_{sj} = \vec{x}_{sj0}$$

$$\frac{1}{\sigma_{V_{ij}}}\frac{\partial V_0}{\partial \vec{X}_c}(\vec{x}_{ci},\vec{x}_{sj})\cdot\vec{e}_c = \frac{V_{ij}}{\sigma_{V_{ij}}}$$

$$\vec{X}_c = \vec{X}_c + \vec{e}_c$$

$$\vec{x}_{ci} = f_i(\vec{X}_c)$$

$$\vec{e}_c < tol\cdot\vec{\sigma}_{c0}$$

$$\frac{1}{\sigma_{V_{ij}}}V_0(\vec{x}_{ci},\vec{x}_{sj})\cdot e_{gj} = \frac{V_{ij}}{\sigma_{V_{ij}}}$$

$$g_j = g_j e_{gj}$$

$$\frac{1}{\sigma_{V_{ij}}}\frac{\partial V_0}{\partial \vec{x}_{sj}}(\vec{x}_{ci},\vec{x}_{sj})\cdot\vec{e}_{sj} = \frac{V_{ij}}{\sigma_{V_{ij}}}$$

$$\vec{x}_{sj} = \vec{x}_{sj} + \vec{e}_{sj}$$

$$\vec{e}_{sj} < tol\cdot\vec{\sigma}_{sj0}$$

$$\vec{X}_c, \vec{x}_{sj}$$

Results: Calibrate Magnetics

- Flux loops give reasonable results
  - Calculated locations mostly within ~2 cm of measured
  - Gains mostly within ~10% of measured
  - χ^2 goodness of fit ~10^{-3}
  - Copper plasma location also found within 1 cm of measured
- Magnetic sensors have very large errors
  - χ^2 goodness of fit too small for floating point
  - Because of poor results, not used in solving for copper plasma location

Theory: Eddy Currents

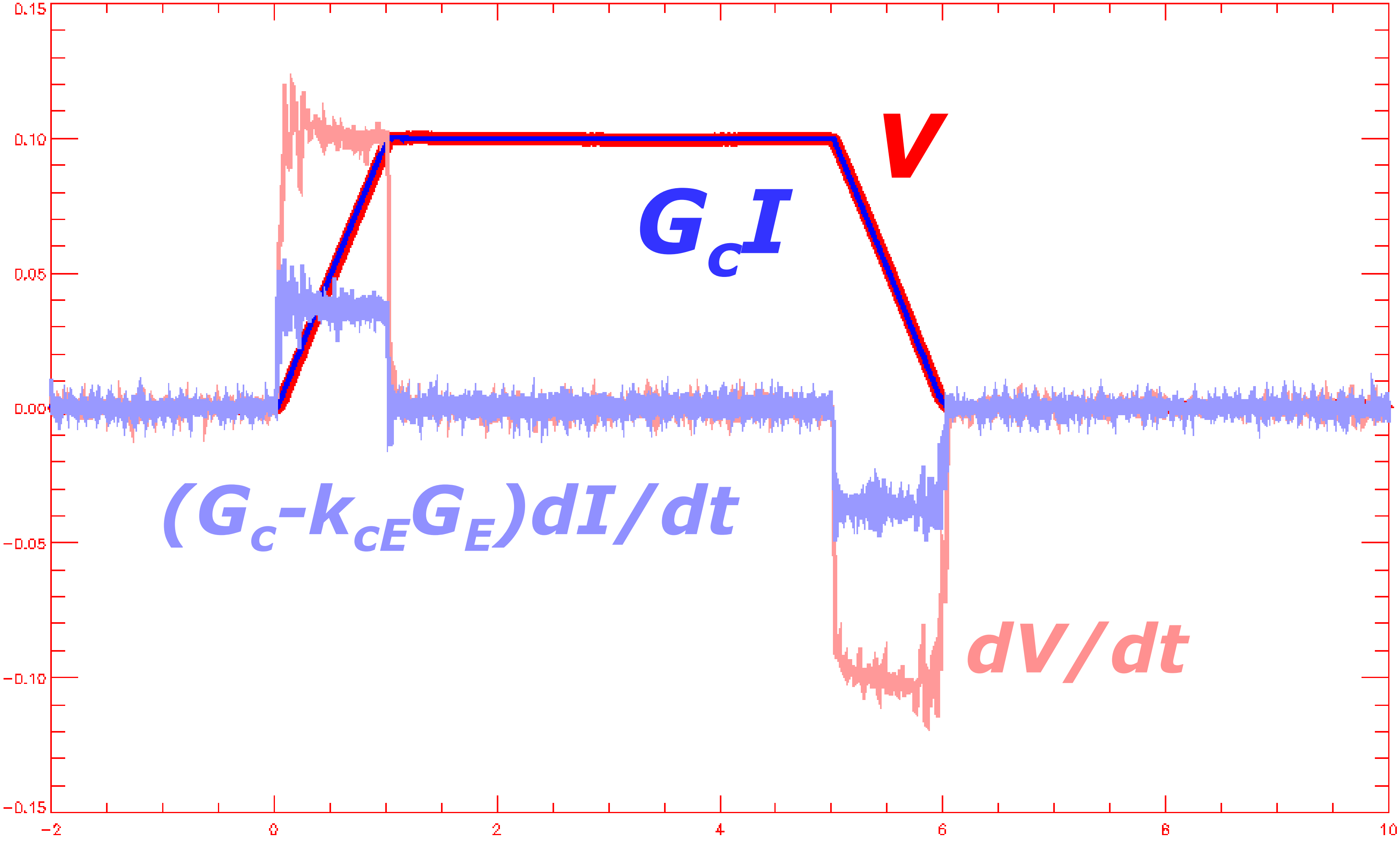
- Measured voltage ***V*** given by sum of all coil and eddy currents ***I*** times their respective Green's functions ***G***
- In reality, many eddy current modes occur
- To simplify, assume one dominant mode per sensor
- Final equation requires no knowledge of eddy currents to calculate decay time, τ_E

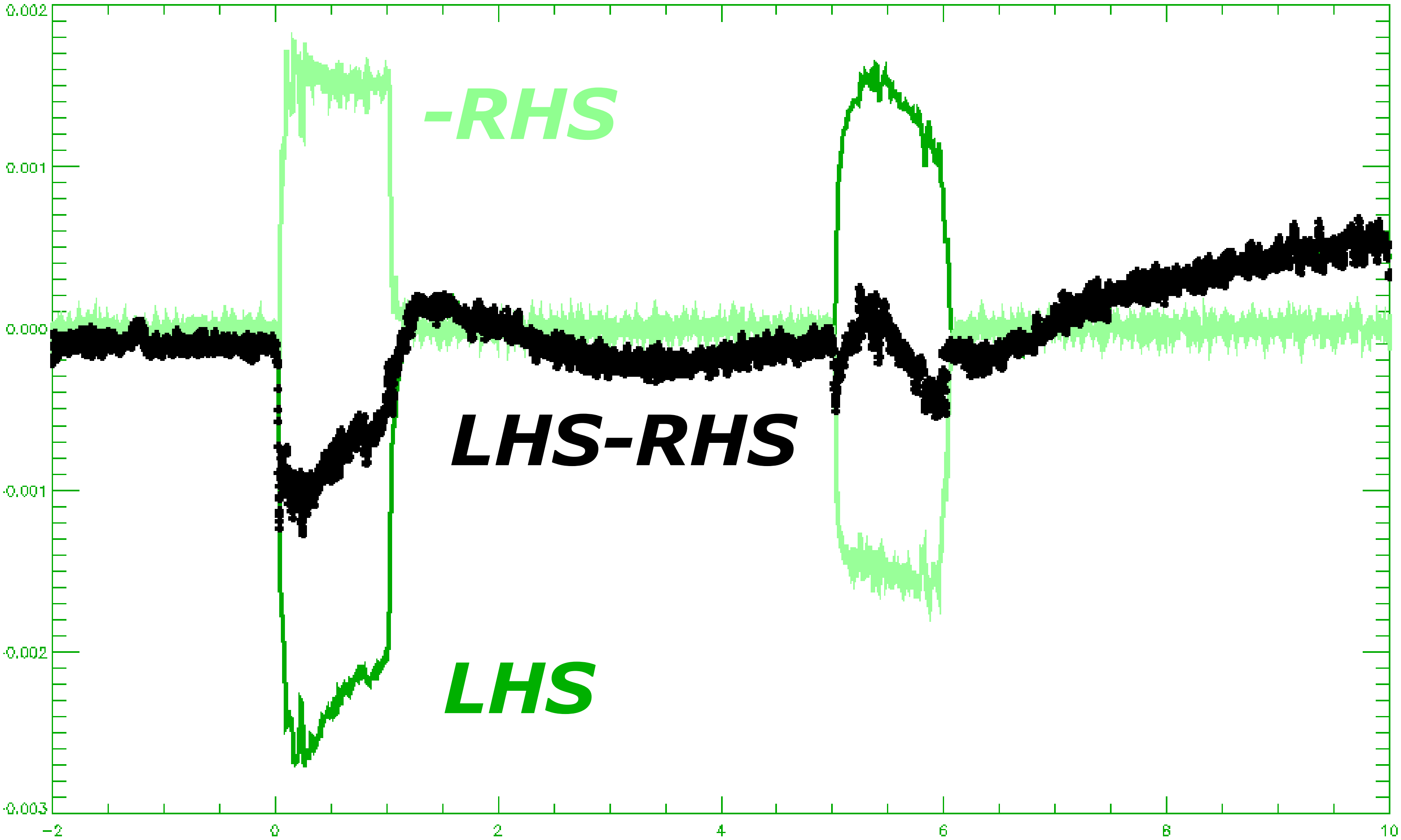
$$(1)\quad V_j = G_{c,j} I_c + \sum_E G_{E,j} I_E \qquad (2)\quad \tau_E \frac{dI_E}{dt} + I_E = k_{cE} \tau_E \frac{dI_c}{dt}$$

$$(3)\quad V_j - G_{c,j} I_c = -\tau_{E,j} \left(\frac{dV_j}{dt} - (G_{c,j} - G_{E,j} k_{cE,j}) \frac{dI_c}{dt} \right)$$

Analysis: Eddy Currents

- Using the ***I*** and ***V*** waveforms from each of the 31 sensors and 36 experiments :
  - Steady state Green's function $G_{c,j}$ is ratio of flattops ***V0*** / ***I0***
    - This is the quantity used in sensor calibration
  - ***d/dt*** dominated Green's function $G_{c,j} - k_{cE,j}G_{E,j}$ is ratio of slopes ***dV/dt / dI/dt*** at some initial time
  - τ_E found by least squares fit to Eq. 3

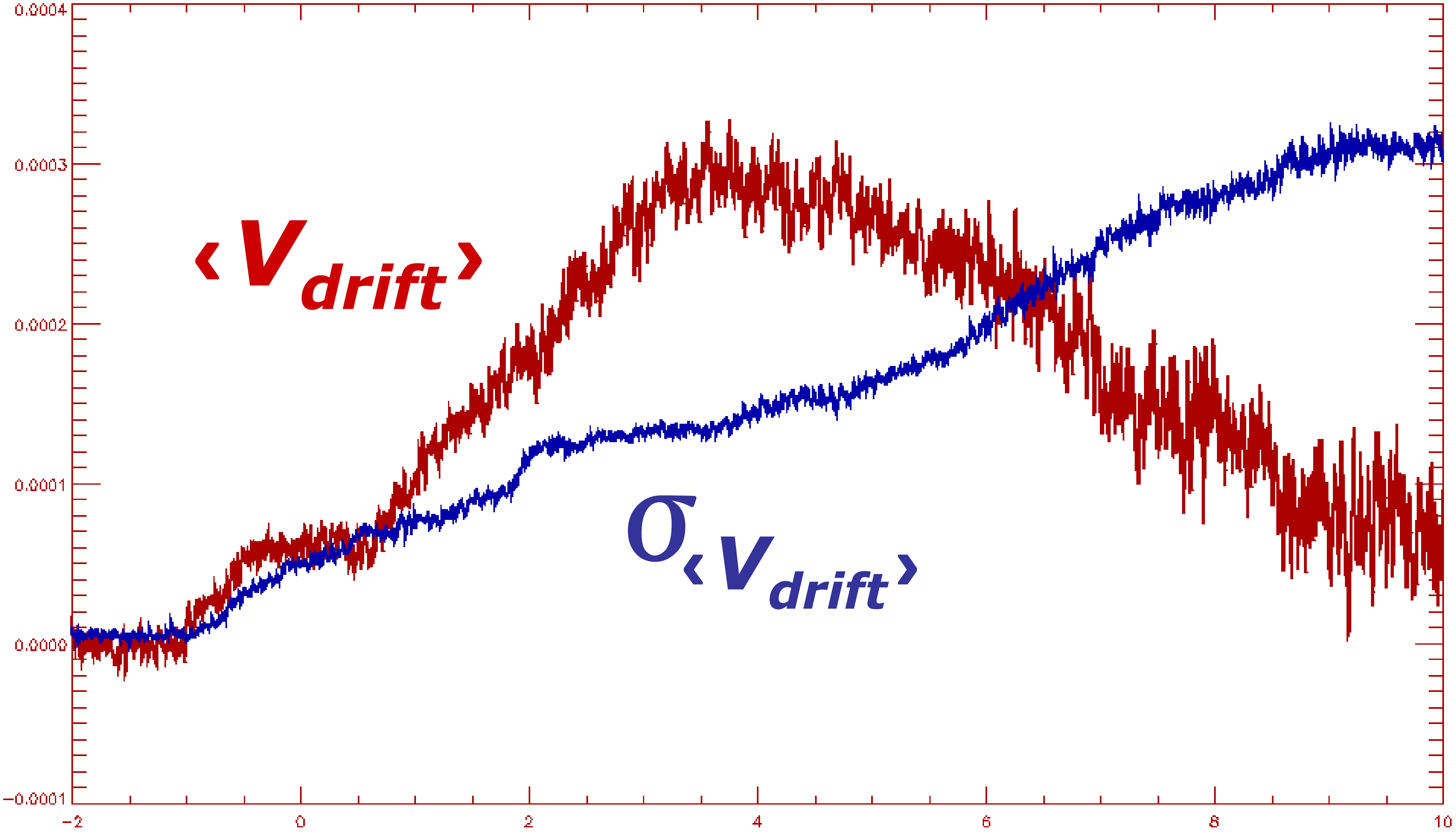

V
G_C I
(G_C-k_CE G_E)dI/dt
dV/dt
0.15
0.10
0.05
0.00
−0.05
−0.10
−0.15
−2
0
2
4
6
8
10



$$V_j - G_{c,j} I_c = -\tau_{E,j}\left(\frac{dV_j}{dt} - (G_{c,j} - G_{E,j} k_{cE,j})\frac{dI_c}{dt}\right)$$

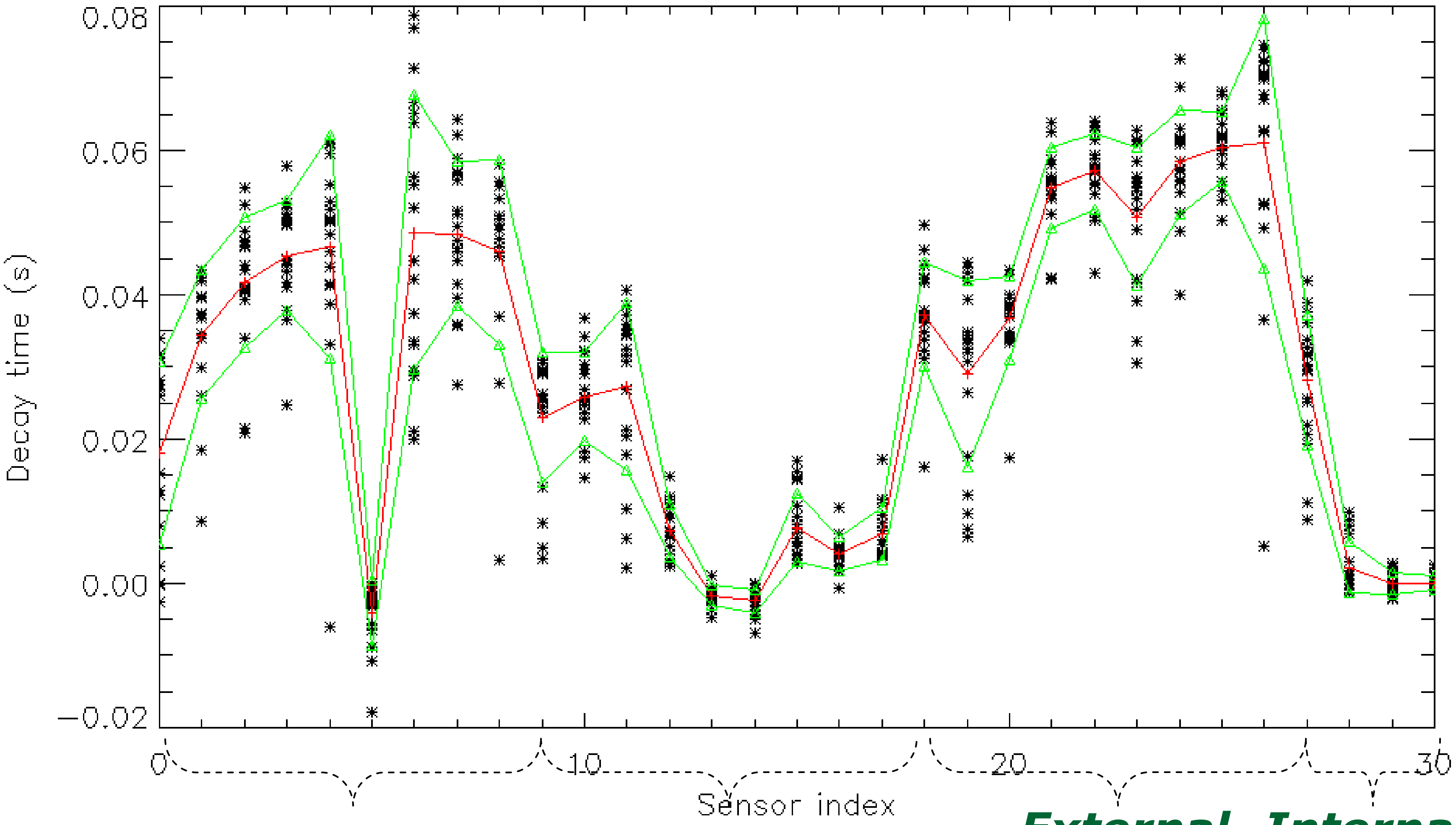
Uncertainty Analysis

- Most uncertainty due to integrator drift
  - Circuits have some drift compensation
  - In analysis, subtract initial drift before pulse
  - Optionally subtract drift during flattop and after pulse
  - Find average drift and uncertainty as a function of time from shots with no current
  - Sum shots with opposite polarity to cancel out stray fields and any reproducible drifts
- Also some uncertainty due to power supply
  - Current is somewhat noisy
    - Difficult to fit initial ***d/dt*** on small, noisy signal
  - Feedback system designed for L-coil, not copper plasma
    - Leads to ringing at corners of trapezoid


$\langle v_{drift} \rangle$
$\sigma_{\langle v_{drift} \rangle}$
0.0004
0.0003
0.0002
0.0001
0.0000
−0.0001
−2
0
2
4
6
8
10

Results: Eddy Currents

- Most sensors in range 30 - 70 ms
  - Standard deviations over all shots 3 - 5 ms
  - Consistent with predictions from theory
- Some < 10 ms or negative, standard deviations as big as τ_E



$B_\perp$

$B_{//}$

External Flux Loops

Internal Flux Loops

Summary

- Sensor calibration and eddy current decay time results generally consistent with previous measurements
- Copper plasma experiment did not yield an improvement in accuracy or precision over previous measurements

Suggestions for Future Work

- Work to reduce integrator drift
- Optimizing power supply for copper plasma could reduce noise and ringing
- Use less simplified model for copper plasma and sensors
  - Multiple windings in copper plasma
  - Possible errors in construction
  - Area and length in magnetic sensors
- Try more robust fitting schemes for calibration
- Improve calculation of $k_{cE,j}G_{E,j}$
  - Use different pulse shape – faster rise time, sinsusoid
  - Use non-linear method to solve for both τ_E and $k_{cE,j}G_{E,j}$
- Do more complete analysis with multiple eddy current modes

Acknowledgements

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- Thanks to the LDX team for assistance in construction and installation of the copper plasma