A Phase Contrast Imaging–Interferometer system for detection of multiscale density fluctuations

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Outline

- Motivation
- Phase Contrast Imaging (PCI)
  - Fundamentals
  - Implementation on DIII-D
  - Response characteristics
- Interferometry
  - Fundamentals
  - Synthetic diagnostic study of low-\(k\) capabilities
  - Low-\(n\) MHD capabilities
- Implementation of a combined PCI–Interferometer on DIII-D
  - Layout and hardware
  - Sound wave cross-calibration
  - Potential upgrades
- Conclusions and future work
Port space and vessel windows will be limited on all future devices – combining diagnostics will be necessary.

Phase contrast imaging (PCI) and interferometry are compatible and complementary:

<table>
<thead>
<tr>
<th>Parameter</th>
<th>PCI</th>
<th>Interferometer</th>
</tr>
</thead>
<tbody>
<tr>
<td>probe beam</td>
<td>single CO₂ beam</td>
<td>single CO₂ beam</td>
</tr>
<tr>
<td>frequency bandwidth</td>
<td>$10 \text{ kHz} &lt; f &lt; 2 \text{ MHz}$</td>
<td>$10 \text{ kHz} &lt; f &lt; 2 \text{ MHz}$</td>
</tr>
<tr>
<td>spatial bandwidth</td>
<td>$1.5 \text{ cm}^{-1} &lt; k &lt; 20 \text{ cm}^{-1}$</td>
<td>$0 &lt; k &lt; 5 \text{ cm}^{-1}$</td>
</tr>
</tbody>
</table>

*All parameters for DIII-D’s currently existing PCI and recently-constructed interferometer*
A combined PCI-Interferometer will allow novel turbulence and MHD investigations on DIII-D

**Turbulence and Transport**: combined system will “fill-out” measured $k$-space; important for model validation

<table>
<thead>
<tr>
<th>$k$</th>
<th>[cm$^{-1}$]</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1.5</td>
</tr>
<tr>
<td>5</td>
<td>20</td>
</tr>
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</table>

**Core MHD**: $n \leq 8$ detected through cross correlation with DIII-D’s existing interferometer ($\Delta \phi = 45^\circ$), allowing studies of toroidal structure and influence on fast particles

**Proof of principle**: ITER and next-step devices will certainly have an interferometer

- Minimal system additions may also allow PCI measurements
Electron density fluctuations modulate the *phase* of electromagnetic waves propagating through a plasma

For a CO$_2$ laser beam ($\lambda_0 = 10.6 \mu$m) in a tokamak plasma, the index of refraction $N$ is

$$N \approx 1 - \frac{1}{2} \left( \frac{\omega_{pe}}{\omega_0} \right)^2$$

Thus, a CO$_2$ beam propagating through a tokamak plasma will acquire a phase shift $\phi$ *relative* to vacuum

$$\phi = \frac{\omega}{c} \int (N - 1) dl = -r_e \lambda_0 \int n_e dl$$

Further, if $n_e = \bar{n}_e + \tilde{n}_e$, there will be a corresponding $\phi = \bar{\phi} + \tilde{\phi}$

$$\tilde{\phi} = -r_e \lambda_0 \int \tilde{n}_e dl$$  \hspace{1cm} (1)
Phase Contrast Imaging (PCI) transforms “invisible” phase modulations into measurable intensity variations.

Plasma fluctuations scatter a portion of the incident radiation

\[ E = E_0 e^{i\phi} \]

but do not alter the resulting intensity

\[ I \propto |E_0 e^{i\phi}|^2 = E_0^2 = \text{const} \]

Delaying unscattered beam by \( \pi/2 \) with a phase plate yields intensity modulations:

\[ E \approx E_0 (1 + i\phi) \]

\[ \Rightarrow E_{\text{PCI}} \approx E_0 (i + i\phi) \]

\[ I_{\text{PCI}} \propto |E_0|^2 (1 + 2\phi) \] (2)
DIII-D’s PCI operates in any tokamak plasma and has high bandwidth, making it a model burning plasma diagnostic.

- CO$_2$ laser is a compromise between high signal and low refraction
- Large bandwidth:

\[
10 \text{ kHz} < f < 2 \text{ MHz} \\
1.5 \text{ cm}^{-1} < k_R < 20 \text{ cm}^{-1}
\]

- Resolves $k_R$ ($\Delta k_R \approx 2 \text{ cm}^{-1}$)
- Localization of high-$k_R$ measurements
The $k_R$ of vertically line-integrated measurements is related to $k_\theta$ via a spatially varying geometric factor

\[
\begin{align*}
R + 2 &- R + 1 \\
R - 2 &- R - 1
\end{align*}
\]

Only fluctuations perpendicular to beam are detected.

PCI’s vertical beam and imaging configuration on DIII-D measure $k_R$

\[
k_R = k_\theta \csc [\alpha(R, z)]
\]

where $\alpha$ is angle between beam and local flux surface.
PCI’s phase plate allows fluctuation detection for $k > k_{\text{min}}$

If the scattered beam falls within the phase groove ($\Delta < d/2$), the signal is cutoff, giving

$$k_{\text{min}} = \frac{k_0d}{2f}$$

On DIII-D, an $f = 80.7''$ mirror focuses the CO$_2$ beam onto a 1 mm phase groove, providing

$$\left( k_R \right)_{\text{min}} = 1.5 \text{ cm}^{-1}$$  \hspace{1cm} (4)

Typical pedestal parameters give

$$k_\theta \rho_s \gtrsim 0.25$$

whereas ITG peaks at $k_\theta \rho_s \sim 0.2$
Low-\(k\) cutoff readily seen in experimental data from PCI

**L-mode**

- \(f\) [kHz]
- \(k_R\) [cm\(^{-1}\)]
- \(150 000\)
- \(t = 2.00s\)

**H-mode**

- \(f\) [kHz]
- \(k_R\) [cm\(^{-1}\)]
- \(150 000\)
- \(t = 2.60s\)

[Image of experimental data from PCI showing low-\(k\) cutoff]

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**DIII-D**

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The plasma leg undergoes a phase shift $\phi = \phi(r, t)$, and the resulting electric field at the detector is

$$E_{\text{det}} = E_R + E_P e^{i\phi}$$

with corresponding intensity

$$I_{\text{det}} = E_R^2 + E_P^2 + 2E_R E_P \cos \phi$$

Interferometer with magnification $M$ measures fluctuations

$$0 \leq k \leq \frac{2\pi M}{s}$$

With $s = 1\, \text{mm}$ and $M = 0.08$

$$0 \leq k_R \leq 5.0\, \text{cm}^{-1} \quad (5)$$

complementing PCI’s $k$-range
Synthetic diagnostics and GYRO simulations used to model PCI and interferometer response.

Equilibrium Profiles

- $n_e$ [10$^{19}$ m$^{-3}$]
- $T_i$ [keV]
- $T_e$ [keV]
- $r_{\text{min}}$ [m]

Gyro-predicted fluctuations

- $\tilde{n}_e$ [10$^{18}$ m$^{-3}$]
- $t$ [ms]

$\tilde{n}_e$ cross section

PCI beam $r_{\text{min}} = 0.375$ m, $\theta = 0$
Synthetic diagnostics confirm that interferometry’s low-$k$ detection *complements* PCI’s high-$k$ capabilities.
Toroidally spaced interferometers allow novel low-\(n\) mode studies; applications to fast particle transport

Cross-correlating signals from toroidally spaced interferometers \((\Delta \zeta = 45^\circ)\) allows low-\(n\) toroidal mode identification

\[
  n = \left( \frac{2\pi}{\Delta \zeta} \right) f_\tau = 8 f_\tau
\]

where \(f\) is the mode frequency and \(\tau\) is the time delay
IMPLEMENTATION of a combined PCI-Interferometer on DIII-D
Interferometry and PCI can be *simultaneously* implemented with minimal optical table changes and no port changes.

- Acousto-optic modulator (AOM) frequency shifts the reference beam by 27 MHz, allowing heterodyne detection.
- Two-color detection is *not* required to measure *fluctuations*.
Cooled interferometer detector increases signal-to-noise ratio (SNR), minimizing power diverted from PCI system

VIGO PVM-2TE-10.6 used as interferometer detector

- Photovoltaic operation
- 2-stage thermoelectric cooling
- $D^* \geq 10^8 \text{ cm Hz}^{1/2} / \text{W}$
- Estimated detector SNR:

$$\text{SNR} \equiv \left( \frac{\tilde{\phi}_{\text{meas}}}{\tilde{\phi}_{\text{noise}}} \right)^2 \sim 500 \quad (7)$$

Note: PCI’s LN$_2$-cooled detector ($D^* = 2 \times 10^{10} \text{ cm Hz}^{1/2} / \text{W}$) needed to detect high-$f$, high-$k$ low amplitude signals
Detector measures 27 MHz heterodyne signal that is modulated by phase variations along beam paths.

We have subsequently achieved the expected peak-to-peak signal voltage $V_{pp} \sim 150$ mV.
Analog I/Q demodulation recovers phase $\phi(t)$ from heterodyne measurement.

Phase computed as

$$\phi(t) = \tan^{-1} \left[ \frac{I(t)}{Q(t)} \right]$$ (8)

Demodulator noise figure results in total-system (detector-to-digitizer) signal-to-noise ratio $\text{SNR} \geq 50$
The system response can be empirically characterized by shooting sound waves through the beam path. Using the sound wave dispersion relation $c_s = \frac{2\pi f}{k} \approx 340 \text{ m/s}$, we can determine $k$ from an imposed frequency $f$. 
The combined PCI-Interferometer can detect chirped (1 – 15 kHz) calibration sound waves.

Interferometer detects low-k fluctuations invisible to PCI.

The measurements are complementary.
However, the measured interferometer response differs from theoretical expectations.
Next-steps and potential upgrades to improve interferometer response and explore additional physics

- We are evaluating the effect of the following on the interferometer’s SNR:
  - Matched reference and plasma arm optical path lengths
  - Increased power on interferometer detector
  - Optimized electronics
- $k$-resolved measurements via interferometer detector array
  - Electron density gradient fluctuation measurements via differential interferometry
- Equilibrium and fluctuation measurements via:
  - Two-color (e.g. CO$_2$-HeNe) interferometry, or
  - Dispersion interferometry
- Radially viewing PCI–interferometer for pure $k_\theta$ detection
Conclusions

- PCI and interferometry are compatible and complementary reactor-relevant diagnostics that already inform compelling physics investigations on today’s devices.

- A combined PCI–interferometer on DIII-D will allow:
  - Multiscale $\bar{n}_e$ measurements ($0 \leq k_R \leq 20 \text{ cm}^{-1}$)
  - Low-$n$ MHD studies
  - Diagnostic proof-of-principle

- A combined PCI–interferometer has been constructed at DIII-D
  - Small changes to the pre-existing PCI optical table have allowed heterodyne interferometric measurements
  - Cross-calibration sound wave measurements have confirmed the system’s multiscale capabilities
  - Interferometer SNR and response are being optimized
  - Physics data expected at start of DIII-D’s 2015 campaign