Convective Cell Formation in a Levitated Dipole

J. Kesner

Plasma Science and Fusion Center

Massachusetts Institute of Technology

Cambridge, MA 02139

D.T. Garnier

Columbia University

New York, NY 10027

December, 1999

Abstract

A plasma that is confined in a closed field line geometry such as a levitated dipole may be subject to the formation of convective cells. Using a "Drift" fluid model we show that such flow patterns can represent an appropriate equilibrium in a dipole field that contains a plasma with a non-axisymmetric pressure profile. This can provide a means of fueling and heating a dipole confinement device.

MIT Report PSFC/JA-99-35

The use of a dipole magnetic field generated by a levitated ring to confine a hot plasma for fusion power generation was first considered by Akira Hasegawa [1, 2]. As a laboratory approach to controlled fusion a dipole configuration would generate the magnetic field with a circular magnet internal to the plasma. To avoid losses on supports the ring would need to be superconducting and it would be magnetically levitated within the vacuum chamber. Since a large flux expansion is necessary to obtain a fusion grade plasma for a fixed edge plasma the configuration requires a small coil that is levitated within a relatively large vacuum chamber. An initial test of this concept is embodied in the Levitated Dipole Experiment (LDX) which is being built jointly by Columbia University and MIT [3].

Since electric potential tends to be constant along magnetic field lines it can vary both radially and azimuthally in a closed field line system. This can lead to the formation of convective cells in closed field line configurations that lack magnetic shear [4, 5, 6]. Thus in a dipole configuration the magnetic field may be axisymmetric but field lines can charge up leading to steady state, non-axisymmetric $\vec{E} \times \vec{B}$ flow patterns.

The conceptual view of convective cells, presented by Dawson and Okuda [4] is that in a plasma, stable modes will be excited by thermal fluctuations which, even in the absence of equilibrium flows, will develop non-linearly, into convective cells. Convective cells were observed in closed field line experiments (for example Ref. [6]) but the effective temperature was observed to be several orders of magnitude above the thermal level. In this work we show that when the plasma heating is non-axisymmetric, convective flows tend to develop in the equilibrium. In solving the equilibrium problem it is important to include the diamagnetic flows which are of comparable magnitude to the $E \times B$ (electric-field driven) flows. (The $E \times B$ flow is the only flow included in the magnetohydrodynamic (MHD) equations). In fact convective cells are seen to form in regions where the $E \times B$ and diamagnetic flows approximately cancel.

It should be pointed out that convective cells lead to particle transport but not necessarily to energy transport. In fact at the pressure gradient that is marginally stable to interchange modes convective flows would transport particles without a net transport of energy, i.e. the hot core plasma cools as it convects outwards and the outer plasma heats as it convects inward. This would provide an ideal approach for the fueling and heating of a reacting fusion plasma.

The MHD equations often provide a convenient starting point for a fluid model. However, MHD assumes that $E \times B$ drifts are much larger than diamagnetic drifts and this assumption is not appropriate for the study of convective flows. We will therefore use the so-called "drift model" [7, 8] which keeps both flow terms. In the drift model we approximate the flow by

$$\vec{V} = \frac{\vec{B} \times \nabla \phi}{B^2} + \frac{\vec{B} \times \nabla p}{e n_e B^2} + V_{\parallel} \vec{b} \equiv \vec{V}_E + \vec{V}_P + V_{\parallel} \vec{b} \tag{1}$$

with $\vec{b} \equiv \vec{B}/B$ and we assume a single hydrogenic ion species, i.e. $n_e = n_i$ is the plasma density. The lowest order moments of the drift model fluid equations take the form:

$$\nabla(\rho \vec{V}) = S \tag{2}$$

$$\rho \vec{V} \cdot \nabla \vec{V}_E + \rho(\vec{V}_E + V_{\parallel} \vec{b}) \cdot \nabla \vec{V}_{\parallel} = -\nabla p + \vec{J} \times \vec{B} \tag{3}$$

where $\rho(\psi)$ is the mass density, $\rho = m_i n(\psi)$, p is the pressure, S is the particle source and J and B are the self-consistent current density and magnetic field. The Eqs. (2) and (3) are respectively the continuity and the force balance equations. Notice the momentum equation is similar to the MHD force balance equation except for $\vec{V} \rightarrow \vec{V}_E$ in the advective term due to the so-called gyroviscous cancellation [7, 8].

For the field of a floating ring (at finite β) the magnetic field is poloidal and to lowest order axisymmetric and we can use orthogonal flux co-ordinates $(\psi,\ \zeta,\ \chi)$ such that $\vec{B} = \nabla\psi \times \nabla\zeta = \alpha \nabla \chi$ with ψ the flux co-ordinate, ζ the toroidal angle co-ordinate and χ the along-the-field-line coordinate. (For a vacuum field $\alpha = 1$ and χ is the magnetic scalar potential.) In equilibrium the pressure and the electrostatic potential tend to be constant on a field line and when heating is not azimuthally symmetric in a closed field line system, the pressure can vary both radially (in the flux co-ordinate) and azimuthally. Since we can write $\vec{V} \cdot \nabla V = \nabla V^2/2 - \vec{V} \times (\nabla \times \vec{V})$, we expect an azimuthal gradient in p to lead to an azimuthal gradient in $V^2/2$.

In flux co-ordinates the flow velocity (Eq. 1) becomes

$$\vec{V} = \left(\frac{\nabla \phi \cdot \nabla \zeta}{B^2} + \frac{\nabla p \cdot \nabla \zeta}{e n_e B^2} \right) \nabla \psi - \left(\frac{\nabla \phi \cdot \nabla \psi}{B^2} + \frac{\nabla p \cdot \nabla \psi}{e n_e B^2} \right) \nabla \zeta + V_{\parallel} \vec{b} \tag{4}$$

which can be written as

$$\vec{V} = \frac{1}{RB} \left(\frac{\partial \phi}{\partial \zeta} + \frac{1}{e n_e} \frac{\partial p}{\partial \zeta} \right) \vec{e}_{\psi} - R \left(\frac{\partial \phi}{\partial \psi} \vec{e}_{\zeta} + \frac{1}{e n_e} \frac{\partial p}{\partial \psi} \right) \vec{e}_{\zeta} + V_{\parallel} \vec{b} \tag{5}$$

with $\vec{e}_{\psi} = \nabla \psi / |\nabla \psi|$ and $\vec{e}_{\zeta} = \nabla \zeta / |\nabla \zeta|$. We will assume up-down symmetry for the floating ring field and plasma.

We are interested in the ϕ contours on the midplane defined by $\chi = 0$. We will simplify Eq. (3) by assuming that $V_{\parallel} \partial / \partial \chi = 0$ on the midplane and that $\vec{\kappa}_{\psi}$ is not changed by the perturbed (non-axisymmetric) pressure. This assumption decouples the continuity equation from the momentum equation at the midplane. The variation of $V_{\parallel}$ is discussed in the appendix. Setting $V_{\parallel} \partial / \partial \chi = 0$ and taking the dot product of Eq. (3) with $\vec{e}_{\psi}$ and $\vec{e}_{\zeta}$ yields the following momentum balance equations:

$$\begin{aligned} \rho \left((\frac{\partial \phi}{\partial \zeta} + \frac{1}{e n_e} \frac{\partial p}{\partial \zeta}) \frac{\partial}{\partial \psi} - (\frac{\partial \phi}{\partial \psi} + \frac{1}{e n_e} \frac{\partial p}{\partial \psi}) \frac{\partial}{\partial \zeta} \right) \frac{1}{RB} \frac{\partial \phi}{\partial \zeta} - \rho R (\frac{\partial \phi}{\partial \psi})^2 - \frac{m_i R}{e} \frac{\partial \phi}{\partial \psi} \frac{\partial p}{\partial \psi} \\ = -\vec{e}_{\psi} \cdot \nabla (p + B^2/2\mu_0) + B^2 \vec{\kappa}_{\psi} / \mu_0 \end{aligned} \tag{6}$$

and

$$\begin{aligned} \rho \left((\frac{\partial \phi}{\partial \psi} + \frac{1}{e n_e} \frac{\partial p}{\partial \psi}) \frac{\partial}{\partial \zeta} - (\frac{\partial \phi}{\partial \zeta} + \frac{1}{e n_e} \frac{\partial p}{\partial \zeta}) \frac{\partial}{\partial \psi} \right) R \frac{\partial \phi}{\partial \psi} + \frac{\rho}{RB} \frac{\partial \phi}{\partial \psi} \frac{\partial \phi}{\partial \zeta} + \frac{m_i}{eRB} \frac{\partial p}{\partial \psi} \frac{\partial \phi}{\partial \zeta} \\ = -\frac{1}{R} \vec{e}_{\zeta} \cdot \nabla (p + B^2/2\mu_0) \end{aligned} \tag{7}$$

with $\vec{\kappa} = \vec{b} \cdot \nabla \vec{b}$ and for the poloidal field of a floating ring $\vec{\kappa} = \kappa \vec{e}_\psi = \vec{\kappa}_\psi$. Equations (6) and (7) represent momentum balance in the $\nabla\psi$ and $\nabla\zeta$ directions. We have used $(\partial/\partial\zeta)\vec{e}_\psi = \vec{e}_\zeta$ and $(\partial/\partial\zeta)\vec{e}_\zeta = -\vec{e}_\psi$.

We will consider the case in which the equilibrium is, to lowest order, axisymmetric and for simplicity consider a low β equilibrium (although a high β equilibrium could have also been used). We consider plasma flows in the presence of a small non-axisymmetric heating component which would lead to a non-axisymmetric pressure profile. Therefore we will assume pressure, potential and magnetic field profiles that contain small harmonic azimuthal variations as follows:

$$
\begin{aligned}
p(\psi,\zeta) &= p_0(\psi) + p_1(\psi) cos(n\zeta) \\
\phi(\psi,\zeta) &= \phi_0(\psi) + \phi_1(\psi) cos(n\zeta) \\
\vec{B}(\psi,\zeta,\chi=0) &= \vec{e}_Z[B_0(\psi) + B_1(\psi) cos(n\zeta)]
\end{aligned} \tag{8}
$$

with $\phi_1/\phi_0 \equiv \Delta \ll 1$, etc. and linearize equations (6) and (7). At the midplane ($\chi = 0$) we can express the linearized force balance equations as ordinary differential equations (ODEs) in the midplane radius, R, noting that $d\psi = -BRdR$ and $\vec{B} = B\vec{e}_Z$. Thus we obtain the momentum balance equations (at the midplane):

$$
R\phi'\left(2\phi_0' + \frac{p_0'}{en_e}\right) - n^2\phi\left(\phi_0' + \frac{p_0'}{en_e}\right) + R\phi_0'\frac{p'}{en_e} = \frac{(RB_0)^2}{\rho}\left((p' + \frac{B_0B_{1Z}' + B_0'B_{1Z}}{2\mu_0}) - 2\vec{\kappa}_R\frac{B_0B_{1Z}}{\mu_0}\right) \tag{9}
$$

and

$$
\phi'\left(\phi_0' + \frac{p_0'}{en_e}\right) - \phi\left(\frac{\phi_0'}{R} + \frac{p_0'}{en_eR} + \phi_0'' - \phi_0'\frac{dlnB_0}{dR}\right) + \frac{p}{en_e}(\phi_0'\frac{dlnB_0}{dR} - \phi_0'') = -\frac{B_0^2}{\rho}\left(p + \frac{B_0B_{1Z}}{\mu_0}\right). \tag{10}
$$

We have dropped the subscripts on ϕ_1 and p_1 and the prime denotes derivatives with respect to the midplane radius. Equations (9) and (10) are coupled ODE's. Using a guess for $p(R)$, $p_0(R)$, $\rho_0(R)$ and $\phi_0(R)$ we can solve for ϕ and B_{1Z}. Appropriate boundary conditions are $\phi(R_0) = \phi(R_w) = 0$ with R_0 the radial location of the outer edge of the floating coil and R_w the location of the wall of the vacuum vessel. We solve using a shooting method in which we set $\phi(R_0) = 0$ and adjust $B_1(R_0)$ to obtain $\phi(R_w) = 0$

From Eq. 9 we observe that when $|\phi_0' + p_0'/en_e| \ll |\phi_0'|$, which corresponds to a region where the equilibrium fluid flow nearly stagnates, we can estimate $eR\phi'/T \sim (R/\rho_{Li})^2\Delta \gg 1$ with ρ_{Li} the ion gyroradius. In this circumstance ϕ' can become large and if $|\phi'| > |\phi_0'|$ the linearity assumption is violated in this region. The linearization, however, remains appropriate everywhere except for the small region where the fluid flow stagnates (near where $\phi_0' + p_0'/en_e \sim 0$) and the solution in the stagnation region must connect smoothly to the linear solutions on both sides of this region. We are primarily interested in determining the width of the region in which $|\phi'| > |\phi_0'|$. Therefore we will assume that the solution in the stagnation region is approximately correct.

Importantly, when $|\phi'| > |\phi_0'|$ and $\phi'/\phi_0' < 0$ the ExB guiding center drift will reverse and result in the formation of a closed guiding center flow pattern which corresponds to a convective cell. Since the diamagnetic drift does not represent a movement of guiding centers it does not contribute to the convective cell formation.

When $\phi_0' + p_0'/en_e = 0$ we find that $|\phi'| \to \infty$. However the validity of the fluid equations requires the assumption of small ion gyro radius, i.e. $|e\phi'| < T_i/\rho_{Li}$. We will choose equilibrium profiles that avoid the condition $\phi_0' + p_0'/en_e = 0$ and thereby satisfy the small gyroradius ordering.

We will consider a plasma confined in the field of a floating ring. The magnetic field is only poloidal and the closed field lines (in the poloidal plane) link the current loop so that the plasma confined in this configuration will surround the "floating" coil. We will utilize the magnetic field of a circular current loop which can be expressed in terms of elliptic functions and for illustrative purposes we utilize plasma parameters similar to those expected in the LDX experiment [3] (loop radius is 0.4 m). The use of the vacuum field of a current loop implies a low beta plasma. The assumption of low beta is not necessary to the calculations that follow but it is convenient to use this simple field representation. We also assume an equilibrium azimuthal rotation, i.e. $\omega_\zeta = V_\zeta/R$ and for simplicity we will consider the lowest order rotation to be rigid, i.e. ω_ζ =constant.

We expect that non-axisymmetric heating will result in a non-axisymmetric pressure profile. However, due to the toroidal rotation the pressure asymmetry is expected to be smaller than the heating asymmetry, i.e. a heat deposition $H(\psi,\zeta) = H_0(\psi) + \delta H(\psi,\zeta)$ yields a pressure profile $p(\psi,\zeta) = p_0(\psi) + \delta p(\psi,\zeta)$, and $\delta p/p = (1/\tau_E\omega_\zeta)\delta H/H$ and $\tau_E = a^2/4\chi_E$ with χ_E is the thermal diffusivity.

Consider a model pressure profile on the outer midplane, $p_0(R)$, which will have $p_0 = 0$ at the outer radius of the floating ring (at $R = R_0$) and at the vacuum chamber wall (at $R = R_w$):

$$\begin{aligned}
p_0(R) &= 0.5P_0\left(1 - cos(2\pi n\frac{R-R_0}{R_w-R_0})\right)^{10} \\
\rho(R) &= m_i n_e(R) = 0.5\rho_0\left(1 - cos(2\pi n\frac{R-R_0}{R_w-R_0})\right)^{8} \qquad (11)
\end{aligned}$$

For this model profile the maximum value of $p(R)$ occurs at $(R_w + R_0)/2$. The perturbed pressure, $p_1(R) = \Delta\ p_0(R)$ and Δ the ratio of the non-axisymmetric pressure to the background pressure.

The assumed equilibrium pressure and density profiles are shown in Fig. 1. The pressure extends from R=0.5 m, assumed to be the edge of the floating coil, out to R=2.5m , the assumed wall location and has an assumed peak at R=1.5 m. We will also impose an equilibrium potential profile that gives rise to the equilibrium potential profile shown in Fig. 1c which corresponds to a rigid plasma rotation of 1.3 KHz.

The net equilibrium flow, $V_E + V_P$ (proportional to $\phi_0' + p_0'/en_e$) is shown in Fig. 2. Since the pressure gradient changes sign on the two sides of the pressure peak the two flow terms will subtract in the outer part of the plasma beyond the pressure peak. For our assumed profiles the net equilibrium flow is close to zero at R=1.7 m.

For a perturbed density profile, $p(R) = 0.02p_0(R)$, an n=1 azimuthal pressure variation and an equilibrium potential profile characterized by rigid rotation at 1.3 KHz with an edge potential of 100 eV, the resulting radial perturbed potential variation is displayed in Fig. 3. The maximum value of ϕ' shown in Fig. 3 is 10 V/cm which occurs at the position of the minimum of $\phi_0' + p_0'/en_e$. The requirement for validity of the fluid equations is $\phi' < T_i/\rho_i$ and if we take T_i=100 eV and B=0.2T we obtain the requirement $|\phi'| < 200$ V/cm, which is easily satisfied. The perturbed potential is maximum at R=1.6 m. The midplane contours of perturbed potential are shown in Fig. 4 and we observe the formation of an island near R=1.6 m, i.e. on the outside of the peak pressure location. Since the diamagnetic flow is not a guiding center flow the contours of perturbed potential can be thought of as streamlines of the flow of gyrocenters. Thus we find that convective cells tend to form in the region where $\phi_0' + p_0'/en_e \ll \phi_0'$.

Strictly speaking, the equilibrium curvature and magnetic field gradient drifts are guiding center drifts and should be considered in determining the pattern of guiding center drifts. For simplicity we could consider the equilibrium toroidal drift, which we took to represent a rigid rotation, to represent the sum of curvature-driven and electrostatic potential driven drifts.

We have considered a situation in which the asymmetric heating had a similar radial pressure profile to the equilibrium (symmetric) pressure. In the region between the pressure peak and the vacuum chamber wall we expect the pressure profile to be marginally stable to interchange modes, i.e. $p \propto V^{-\gamma}$ with $\gamma = 5/3$ and $V = \oint d\ell/B$. In so far as the flow is adiabatic (as assumed in MHD), energy is not expected to be transported by flows when the marginal interchange condition are satisfied. Thus convective flow in the outer region of the plasma (beyond the pressure peak) are expected to transport particles without transporting energy [9]. This would offer an ideal means for fuelling, heating and cleansing a dipole-based fusion reactor.

In the inner plasma (between the floating coil and the pressure peak) the electric and diamagnetic flows add and therefore it is difficult for convective flows to form in this region. However, if sufficient asymmetric heating is applied in this inner region convective cells could form and they would result in an undesirable flow of plasma particles and heat towards the ring.

Convective flows can be utilized for plasma heating and this process illustrates a unique capability of the dipole concept. We have seen that a small amount of pressure asymmetry will generate large scale plasma flow patterns. Assuming adiabatic flow, the plasma energy is conserved as the plasma moves inwards, i.e. pV^γ is conserved, the plasma heats as it moves inwards (and the flux tube is compressed). Therefore a warm edge plasma would heat up as the plasma convects inwards and the convection process provides an ideal approach for the heating of a dipole based fusion power source. As plasma is transported within the moving flux tubes the number of particles within a flux tube is conserved and therefore the density goes as $n_e \propto 1/V$. When $p \propto 1/V^\gamma$ and $n_e \propto 1/V$ the value of $\eta = dln\ T/d\ln\ n_e = 2/3$ which will lead to stability of η driven drift waves [10]. This approach is similar to the suggestion of Hasegawa [1] for the use of low frequency magnetic fluctuations which can break the flux invariant and can result in cross-field transport and associated heating.

The LDX floating dipole experiment is planning to use electron cyclotron resonance (ECR) heating. Although the propagation of ECR waves is limited by a density dependent cutoff ($n_e^{crit} \propto f_{RF}^2$) we observe that if we can heat in the outer, low density plasma region, convective flows may transport the warm plasma inward to effectively extend the applicability of ECR heating. In a dipole experiment the factor $(V_{edge}/V_{core})^\gamma$ can be as large as $10^3 - 10^4$ so a considerable heat-up could be obtained in this fashion. For example one might design a reactor in which the fuel heats up to ignition temperature as it flows inwards and the ash cools as it flows outwards.

Appendix: Parallel Flows

In deriving Eqs. 6 and 7 we have ignored terms that derive from $V_\parallel \vec{b} \cdot \nabla$ in Eq. 3. This is consistent with an assumption that there is up-down symmetry in both the equilibrium quantities and potential ϕ. This does not necessarily imply that there is not flow present along the field and in general $V_\parallel \neq 0$. The parallel momentum equation (Eq. 3) can be written as follows:

$$\vec{B} \cdot \nabla(\frac{V_\parallel^2}{2} + \frac{V_E^2}{2}) + \nabla \cdot (BV_\parallel \vec{V}_E) = -B \cdot \nabla p. \tag{12}$$

In general pressure will vary along the field line. In flux coordinates the continuity equation (Eq. 2) yields:

$$\rho \frac{\partial}{\partial \chi}(\frac{V_\parallel}{B}) = \frac{\partial}{\partial \zeta}(\frac{\rho\alpha}{B^2}\frac{\partial \phi}{\partial \psi}) - \frac{\partial}{\partial \psi}(\frac{\rho\alpha}{B^2}\frac{\partial \phi}{\partial \zeta}) + \frac{\alpha S}{B^2}. \tag{13}$$

Equations 12 and 13 can be solved for pressure, $p(\chi)$ and $V_\parallel$. At the midplane $\partial/\partial\psi = -(1/BR)\partial/\partial R$ and $\partial/\partial\chi = (1/B)\partial/\partial Z$. Equation (13) then becomes:

$$\frac{\partial V_\parallel}{\partial Z} = \frac{B}{R\rho}\frac{\partial \phi}{\partial \zeta}\frac{\partial}{\partial R}(\frac{\rho\alpha}{B^2}) + \alpha S. \tag{14}$$

Acknowledgements

The authors would like to thank M. Mauel and R. Betti for useful insights. This work was supported by the US DoE.

References

[1] A. Hasegawa, Comments Plasma Phys. Controlled Fusion, **1**, 147 (1987) .

[2] A. Hasegawa, L. Chen and M. Mauel, Nucl. Fusion **30**, 2405 (1990)

[3] J. Kesner, L. Bromberg, D.T. Garnier, M.E. Mauel, 17th IAEA Fusion Energy Conf, Paper IAEA-F1-CN-69-ICP/09 Yokohama, Japan (1998), *Fusion Energy 1998*, Yokohama, Japan 1998 (International Atomic Energy Agency, Vienna 1999).

[4] J.M. Dawson, H. Okuda and R. Carlile, Phys. Rev. Lett. **27**, 491 (1971)

[5] H. Goede, D. Humanic, J.M. Dawson, Phys. Fluids **26**, 1812 (1983)

[6] G. Navratil, R.S. Post and A. Butcher Ehrhardt, Phys. Fluids **20**, 157 (1977)

[7] K.V. Roberts and J.B. Taylor, Phys. Rev. Lett. **8**, 197 (1962)

[8] R.D. Hazeltine and J.D. Meiss, "Plasma Confinement", (Addison-Wesley, Redwood City, CA, 1992) 228.

[9] J.M. Dawson, Private communication (1998).

[10] J. Kesner, Phys. Plasmas **5**, 3675 (1998)

Figure Captions

1. Model profiles used in calculation a) density b) pressure and c) rigid rotation equilibrium potential profile.

2. Radial profile of equilibrium net (EXB + diamagnetic) flow.

3. Radial profile of perturbed potential.

4. Midplane contours of perturbed potential.

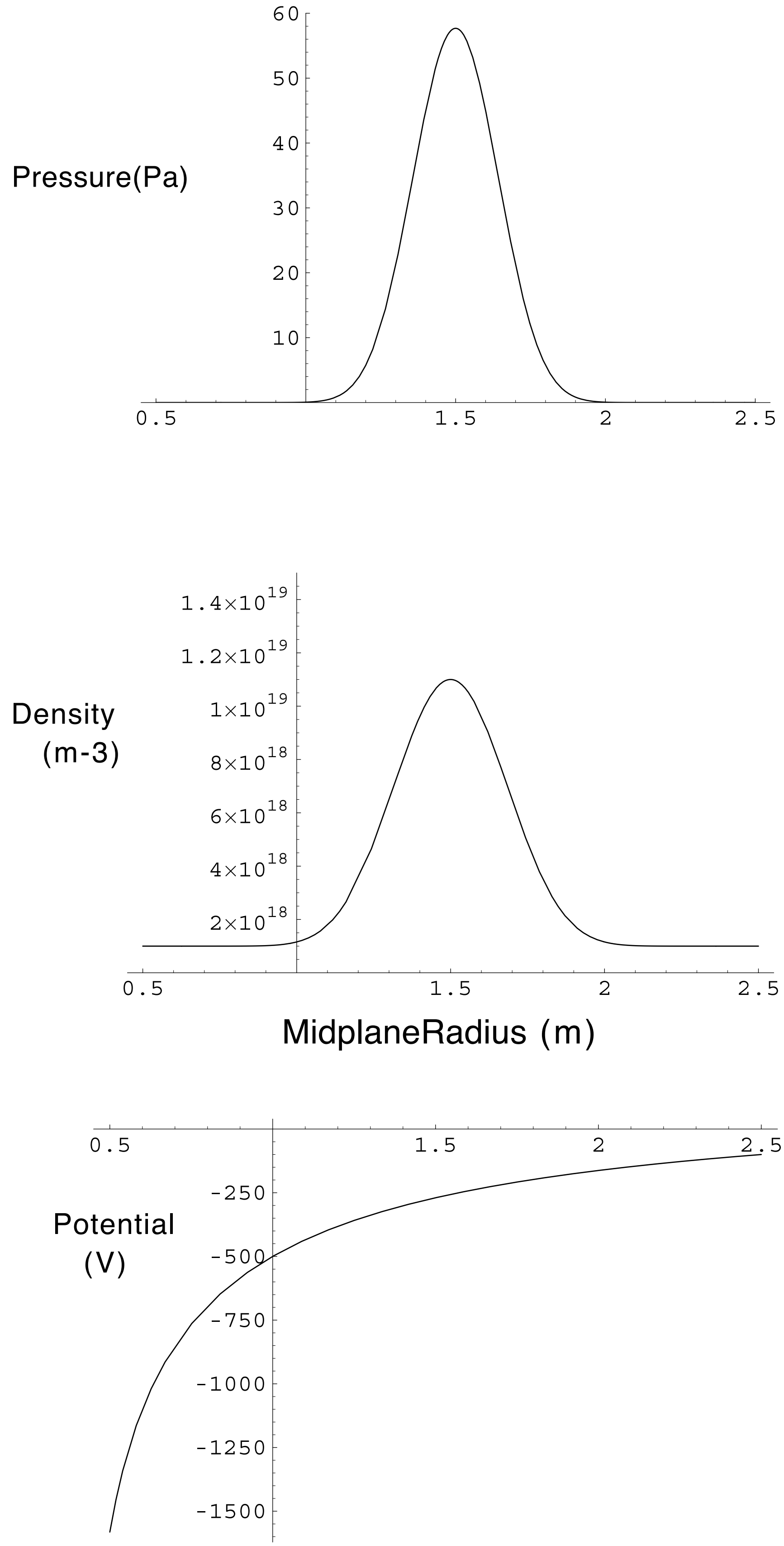

Fig. 1 Model Profiles unse in calculation; a) Pressure, b) density, c) equilibrium electrostatic potential.

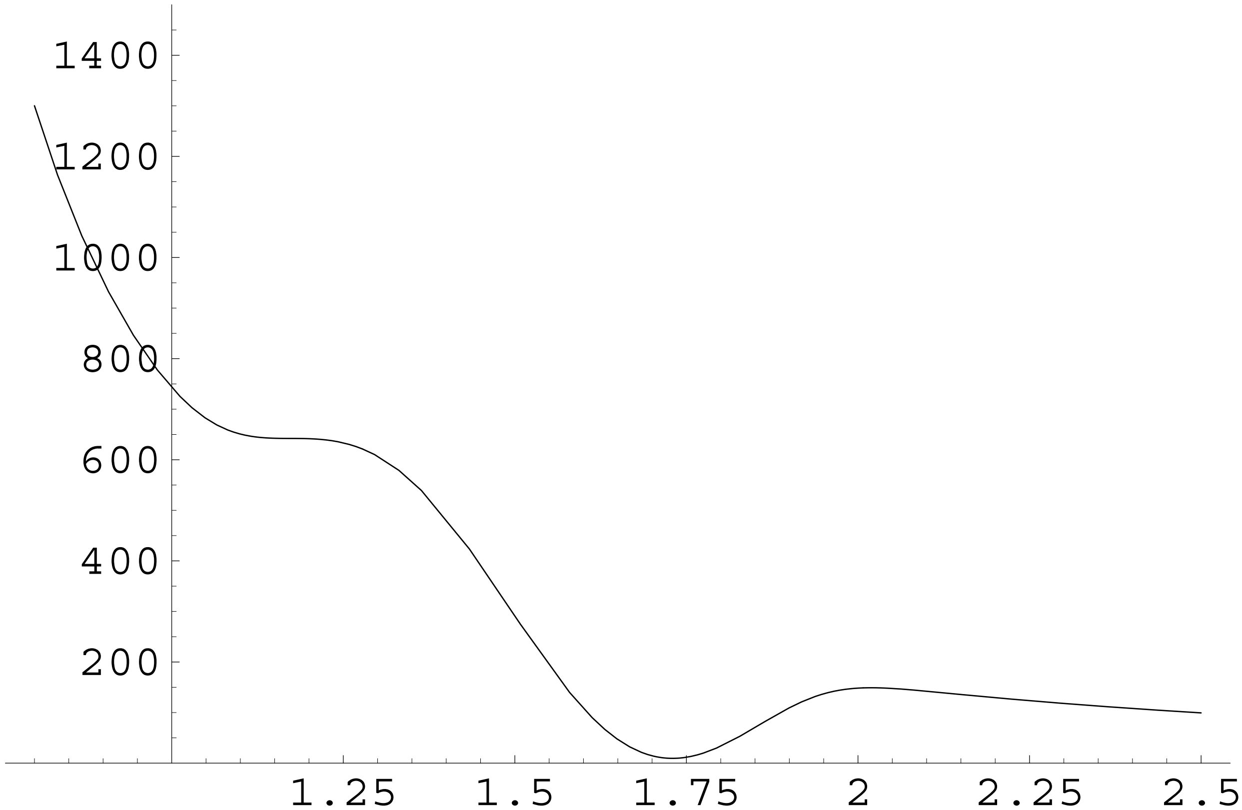

Fig 2. Radial profile of equilibrium (diamagnetic + EXB) flow.

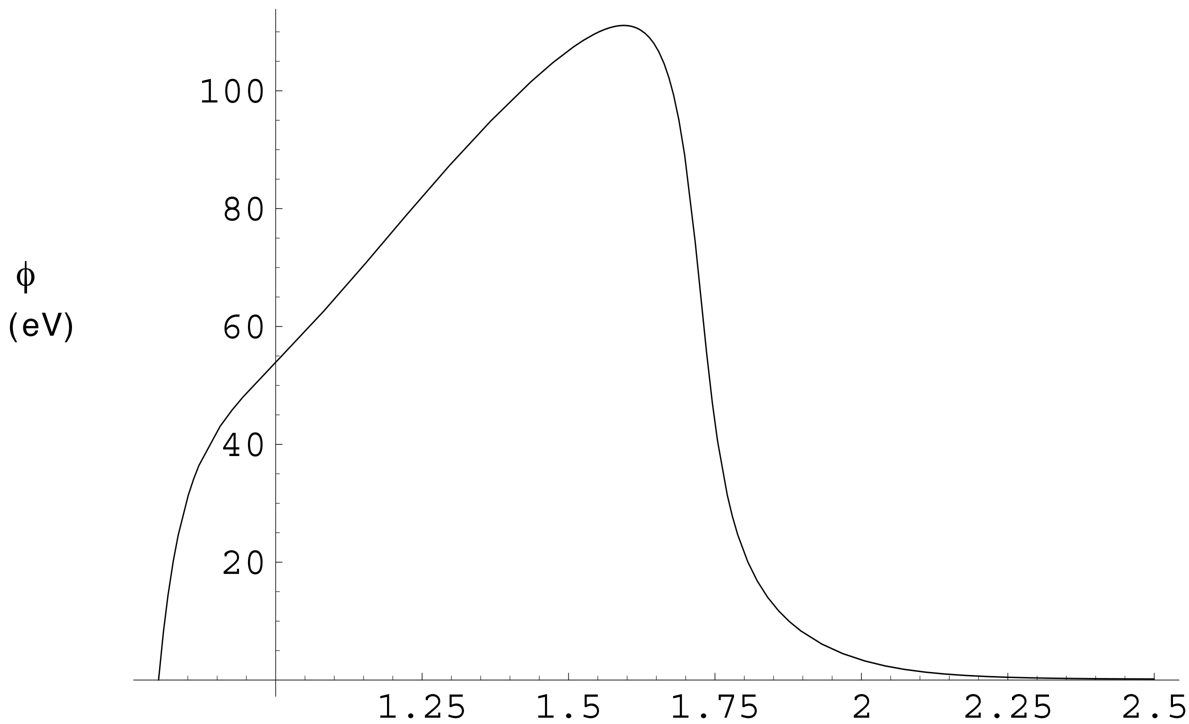

Fig 3. Radial profile of perturbed potential

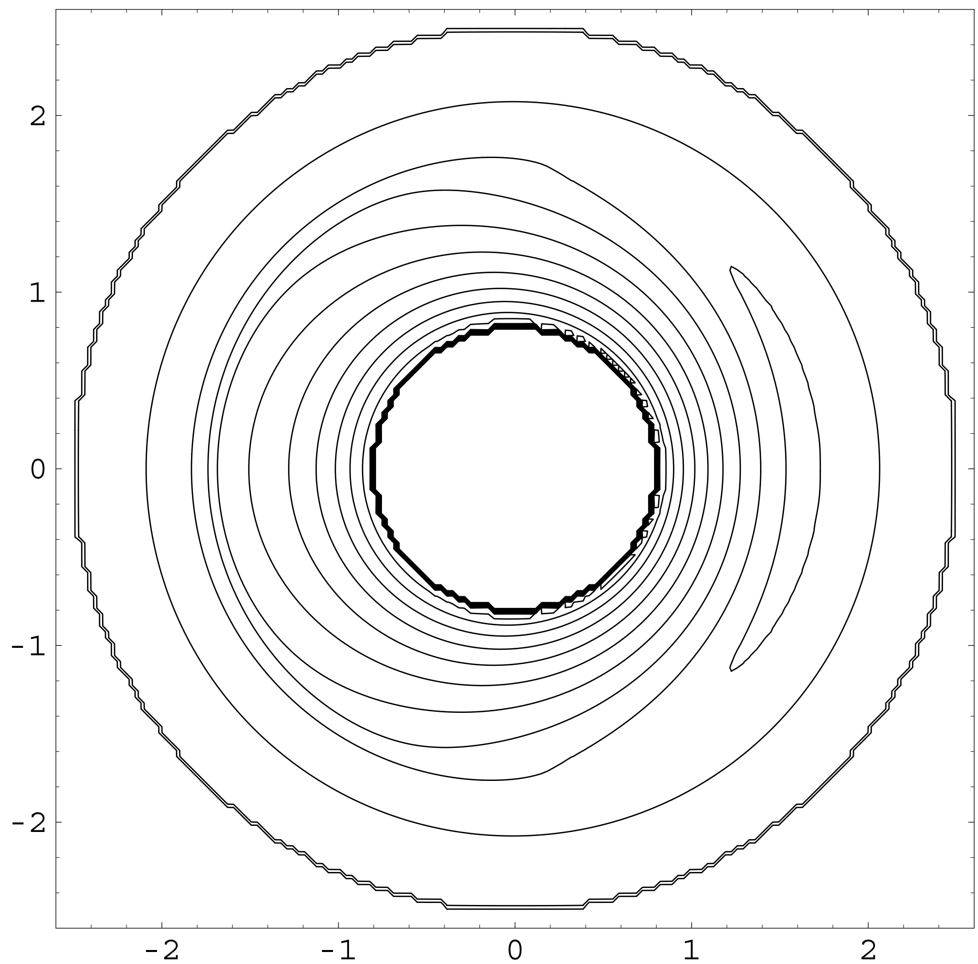

Fig. 4. Midplane Contours of perturbed potential (f=1.3 KHz)