Stability of a plasma confined in a dipole field

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Plasma confined in a magnetic dipole field is stabilized by the expansion of the magnetic flux. The stability of low beta electrostatic modes in a magnetic dipole field is examined when the distribution function is Maxwellian to lowest order. It is shown using a Nyquist analysis that for sufficiently gentle density and temperature gradients the configuration would be expected to be stable to both magnetohydrodynamic and collisionless interchange modes. Furthermore, it is shown that when it is stable to the interchange mode it is also stable to ion temperature gradient and collisionless trapped particle modes, as well as modes driven by parallel dynamics such as the ''universal'' instability.
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I. INTRODUCTION

The use of a dipole magnetic field generated by a levitated ring to confine a hot plasma for fusion power generation was first suggested by Hasegawa.[1] For a magnetic fusion confinement configuration, end losses can be eliminated by levitating the current loop and the resulting configuration possesses uniquely good properties. The coil set is simple and axisymmetric and theory predicts both good confinement properties and a high beta limit. Operation is inherently steady state and the large flux expansion is expected to simplify the divertor design. Since the confining field of a levitated dipole is poloidal there are no particle drifts off the flux surfaces (which in a tokamak leads to a ''neoclassical'' degradation of confinement) and therefore in the absence of turbulent transport confinement could be ''classical.'' Conceptual reactor studies have supported the possibility of a dipole based fusion reactor.[2–4]

It has been conjectured that a plasma confined in a dipole field may be free of the low frequency instabilities[1–5] that are thought to give rise to ''anomalous'' transport in most laboratory plasmas. Hasegawa has pointed out[1,2] that when the plasma is sufficiently collisionless, the equilibrium distribution function may be described by $F_0 = F_0(\mu, J, \psi)$, with μ the first invariant, $\mu = v_\perp^2/2B$, J the second invariant, $J = \oint ds\ v_\parallel$, and ψ the flux invariant. For fluctuations in the range of the curvature drift frequency, flux is not conserved and a collisionless plasma can approach the state $\partial F(\mu,J,\psi)/\partial\psi \rightarrow 0$. Furthermore, when $\partial F(\mu,J,\psi)/\partial\psi = 0$ the plasma can be shown to be stable to drift frequency fluctuations. In a dipole field this condition leads to the prediction that the plasma will be marginally stable when the pressure profiles scale as $p \propto R^{-20/3}$, similar to energetic particle pressure profiles observed in the planetary magnetospheres.[6,7] For fusion relevant plasmas confinement must be maintained on a collisional timescale. Therefore we would expect the distribution function to be, to lowest order, Maxwellian, i.e., $F_0(\mu,J) \rightarrow F_0(\epsilon,\psi)$ and therefore $\partial F/\partial\psi \neq 0$. In this paper we will focus on the stability of drift frequency modes that are driven by $\partial F/\partial\psi$ and are thought to degrade confinement in fusion grade plasmas.

Most toroidal confinement devices with a rotational transform (such as a tokamak) obtain stability from a combination of ''average good curvature'' and magnetic shear. For a plasma surrounding a floating ring, the pressure peak will occur at a distance from the ring surface and beyond the pressure peak the pressure must decrease in a region of ''bad'' curvature. Magnetohydrodynamic (MHD) theory predicts that when a plasma is confined in a ''bad'' curvature region it can be stable provided the pressure gradient does not exceed a critical value. The stabilization derives from the plasma compressibility, i.e., the assumption made in MHD theory that pV^γ is constant. In this paper we will explore the effect of compressibility on drift frequency range modes.

Ideal MHD theory provides a simple approximation for plasma behavior and it does not take account of important ''nonideal'' effects such as finite Larmor radius (FLR) effects, the relationship of density and temperature profiles (characterized by η) or wave particle resonances. One suspects that these nonideal effects may be important in a plasma that is stabilized by compressibility. Goede, Humanic, and Dawson[8] have looked into this question through the application of a particle-in-cell (PIC) code in a slab geometry. They find that stabilization due to compressibility is observed but that nonideal corrections such as finite Larmor radius (FLR) can be important.

The ideal MHD growth rate for unstable interchange modes can be obtained from kinetic theory but the marginal stability condition cannot be simply derived. In a previous study we obtained the dispersion relation from the drift kinetic equation and solve for the stability of several distinct modes.[5] Here we obtain marginal stability by means of a Nyquist analysis which permits us to accurately obtain the marginal stability conditions with a minimum of simplifications. In particular it permits us to include FLR and temperature/density profile effects, wave-particle resonances, collisionality, and parallel dynamics.

In this work we will show that kinetic theory indicates unusual stability properties for a plasma that is stabilized by compressibility. Specifically we will show that while kinetic theory reproduces the MHD result for the stability of interchange modes, it also indicates that both ''trapped particle'' modes and η_i modes will be stable when the interchange modes are stable. Additionally the universal instability is calculated to be stable in a dipole due to the constraints of field line length imposed by the dipole geometry. On the other hand, an η-driven interchange mode can become unstable ($\eta \equiv d \ln T/d \ln n$) in a collisionless plasma when $\eta \neq 1$.

Drift wave theory, as applied to tokamaks, usually assumes an ordering $\omega_* \gg \omega_d$ [ω_* is the diamagnetic drift and ω_d the curvature drift as defined below in Eq. (3)]. The unique property of a dipole to be shown below is that MHD stability requires $\omega_* \lesssim 2\omega_d$ and this ''large plasma ordering'' will be seen to give rise to uniquely favorable stability properties for low frequency drift modes.

In Sec. II we will derive the dispersion relation for low frequency electrostatic modes keeping drift resonant terms. In Sec. III we present stability results derived from a Nyquist solution to the dispersion relation. We first derive the stability condition for collisionless interchange modes which gives a stability criterion that requires that the pressure gradient not exceed a critical value, similarly to the MHD condition. We consider the stabilizing effects of ion finite Larmor radius corrections as well as destabilizing profile effects. We then explore the stability of collisionless trapped particle, collisional electron, and of the universal instability in the large plasma ordering regime.

II. BASIC EQUATIONS

To derive the stability criterion for electrostatic modes we consider a fluctuating potential (ϕ) and ignore any equilibrium electrostatic potential. From Faraday's law it is possible for a perturbation to leave the magnetic field undisturbed if $E = -\nabla\phi$, which is consistent with $\beta \ll 1$. If ϕ varies along a field line, there will be a finite $E_\parallel$ (a situation not possible in ideal MHD theory).

We analyze the stability of such a perturbation under the assumptions that the wave frequency ω is less than the cyclotron frequency Ω_c and that the ion Larmor radius ρ_i is shorter than the perpendicular wavelength $\lambda = 2\pi/k_\perp$ which is, in turn, short compared to a parallel wavelength, $2\pi/k_\parallel$. The appropriate equation for the distribution function $\tilde{f}$ is then[9,10]

$$\tilde{f} = q\phi F_{0\epsilon} + J_0(k_\perp \rho) h, \tag{1}$$

and the nonadiabatic response h satisfies

$$(\omega - \omega_d + i v_\parallel \mathbf{b}\cdot\nabla')h = -(\omega - \omega_*) q\phi F_{0\epsilon} J_0(k_\perp\rho) + iC(h). \tag{2}$$

In Eq. (2) $J_0(k_\perp\rho)$ is the Bessel function of the first kind, $F_0(\epsilon,\psi)$ is the equilibrium distribution function, and

$$F_{0\epsilon} \equiv \frac{\partial F_0}{\partial \epsilon}, \tag{3a}$$

$$\omega_* = \frac{\mathbf{b}\times\mathbf{k}_\perp \cdot \nabla' F_0}{m\Omega_c F_{0\epsilon}}, \tag{3b}$$

$$\omega_d = m\mathbf{k}_\perp \cdot \mathbf{b} \times \frac{(v_\parallel^2 \mathbf{b}\cdot\nabla\mathbf{b} + \mu\nabla B)}{\Omega_c}, \tag{3c}$$

$$\mathbf{B} = \nabla\psi \times \nabla\theta, \tag{3d}$$

$$\mathbf{b} = \mathbf{B}/|\mathbf{B}|. \tag{3e}$$

The gradient ∇' is taken at constant $\epsilon = v^2/2$, $\mu = v_\perp^2/2B$ and θ is the azimuthal angle. We also define $\hat{\omega}_{*p} = k_\perp \nabla p/(n_e \Omega_c)$ and $\hat{\omega}_d = k_\perp T/(R_c \Omega_c)$ with $R_c^{-1} = \mathbf{b}\cdot\nabla\mathbf{b}$.

We consider first a perturbation whose growth time is long compared to a particle bounce time, i.e., modes that conserve both the first ($\mu = v_\perp^2/2B$) and second ($J = \oint v_\parallel dl$) adiabatic invarients. This yields the result that h is a constant along a field line, $h = h_0(\epsilon,\mu,\psi)$. We will determine the constant by taking the bounce average of Eq. (3),

$$h_0 = \frac{-(\omega - \omega_*) q \bar{\phi} F_{0\epsilon} J_0}{(\omega - \bar{\omega}_d + i\nu_q)}, \tag{4}$$

and the overbar indicates a time average:

$$\bar{\phi} = \frac{1}{\tau_B} \oint \frac{dl}{|v_\parallel|} \phi, \tag{5a}$$

$$\tau_B = \oint \frac{dl}{|v_\parallel|}. \tag{5b}$$

For simplicity the collision operator has been replaced by a Krook model in Eq. (4), i.e., $C(h) \to -\nu_j h$ with ν_j the appropriate collision frequency. To proceed further we will assume $|\nabla B|/B \approx 1/R_c$ which is consistent with a low beta approximation.

An approximate form for the bounce average of the curvature drift ω_d in a dipole magnetic field[11] is:

$$\bar{\omega}_d \approx \frac{k_\perp v^2}{\Omega_0 R_{c0}} (0.35 + 0.15 \sin\alpha_0)$$
$$= \frac{k_\perp}{\Omega_0 R_{c0}} (0.5 v_{\perp 0}^2 + 0.35 v_{\parallel 0}^2), \tag{6a}$$

with α_0 the pitch angle at the dipole outer midplane and the subscript ''0'' indicates that quantities are evaluated at the outer midplane (i.e., on the magnetic field minimum). The perpendicular velocity term dominates because the radius of curvature is relatively constant in a dipole and $v_\perp^2/\Omega_c$ is conserved during particle motion, whereas $v_\parallel^2/\Omega_c$ decreases away from the field minimum. We therefore will simplify $\bar{\omega}_d$ as follows:

$$\bar{\omega}_d \to 0.5 \frac{k_\perp v_{\perp 0}^2}{\Omega_0 R_{c0}}. \tag{6b}$$

To obtain the dispersion relationship for electrostatic modes we integrate the perturbed responses over velocity space and apply quasi-neutrality:

$$0 = \sum_j q_j \int d^3 v \tilde{f}_j$$
$$= \sum_j q^2\phi/T_j - q^2\sqrt{2/\pi} \sum_j (\bar{\phi}/T_j) \frac{n_j}{T_j^{3/2}}$$
$$\times \int v^2 dv J_0^2(k_\perp \rho_j) e^{-v^2/2T} \left[\frac{\omega - \omega_{*j}}{\omega - \bar{\omega}_{dj} + i\nu_j} \right], \tag{7}$$

with $\omega_{*j}=\hat{\omega}_{*n}(1-3/2\eta_j+\eta_j v^2/2T)$, $\eta \equiv d(\ln T)/d(\ln n)$, and $\hat{\omega}_{*n}=k_\perp T\nabla n/(n\Omega)$ which is a flux function. The term $\overline{\omega_{dj}}\propto v_\perp^2$ and gives rise to a resonant denominator in Eq. (7).

We can re-derive the MHD growth rate by setting $\nu_j=0$ and expanding the denominator (the compressibility derives from the second order term), assuming $\omega \gg \overline{\omega_d}$. This expansion, however, cannot be made near marginal stability and we will therefore keep the resonance denominator and express the dispersion relation in terms of integrals of the form:

$$\Upsilon(\xi)=\int_0^\infty \frac{e^{-x}dx}{x-\xi}=e^{-\xi}E_1(\xi), \tag{8}$$

with E_1 the exponential integral.[12] Expanding the Bessel functions for small gyro radius ($J_0^2 \approx 1-k_\perp^2 v_\perp^2/2\Omega_{ci0}^2$) we obtain the dispersion relation:

$$f_T \frac{\overline{\phi}}{T}\sum_j \frac{\omega_{*nj}}{\hat{\omega}_{dj}}[(\omega/\omega_{*nj}+\eta_j-1)\Upsilon(\xi_j) - [\eta_j+(k_\perp\rho_j)^2(\omega/\omega_{*nj}+\eta_j-1)](\xi_j\Upsilon(\xi_j)-1) + \eta_j(k_\perp\rho_j)^2(\xi_j^2\Upsilon(\xi_j)-\xi_j-1)]+\frac{2\phi}{T}=0, \tag{9}$$

with $\xi_j=-\omega/\hat{\omega}_{dj}$, $\rho_i^2=T_i/m_i\Omega_{ci0}^2$, $\hat{\omega}_{dj}=k_\perp v_j^2/R_{c0}\Omega_{0j}$, and the thermal speed, $v_j^2=T_j/m_j$. Notice that there is a coupling between FLR and η terms. We will ignore electron FLR terms and assume a two species hydrogenic plasma. The factor f_T will be discussed below in relation to trapped particle modes and $f_T=1$ for interchange modes.

A second approach that was utilized for the solution of the dispersion relation was to approximate $\overline{\omega_d}$ as $\overline{\omega_d}=v^2k_\perp/2\Omega_0 R_{c0}$ and additionally to approximate $J_0^2\approx 1-k_\perp^2 v^2/2\Omega_{ci0}^2$ with $v^2=v_\perp^2+v_\parallel^2$. This leads to a more complicated dispersion relation than Eq. (9) and it was difficult to assess the influence of the approximation that was made for J_0^2. However, it was found that the results from this approximation were not significantly different than those presented below.

In a previous work[5] we examined the stability of several individual modes and appropriate assumptions were made to decouple one mode from another. In this paper we will examine plasma stability using the Nyquist criteria[13] which will allow us to examine stability more generally, including the interaction of different modes. As is well known a Nyquist approach will indicate the stability boundary but it will not yield a frequency or growth rate.

III. RESULTS OF NYQUIST STUDY

A. Collisionless interchange stability

The stability requirement from MHD is $\delta(pV^\gamma)\geq 0$ with $V=\oint dl/B$ and we can therefore define a critical pressure dependence: $p_{\rm crit}\propto 1/V^\gamma$. The critical pressure gradient at the outer midplane is therefore $[\nabla p/p]_{\rm crit}=\gamma[\nabla V/V]_0$ and we observe that $[\nabla V/V]_0\approx 1/R_c$. In a dipole field $V=\oint dl/B \propto R_0^4$ with R_0 the radius on the outer midplane we obtain $p_{\rm crit}\propto R_0^{-20/3}$ and the pressure scale length $r_p^{\rm crit}\equiv p/\nabla p = 0.15\,R_0$. Since, for a dipole the radius of curvature on the outer midplane is $R_{c0}=R_0/3$ we obtain a critical pressure gradient from MHD, namely $R_{c0}\nabla p/p=\hat{\omega}_{*p}/\hat{\omega}_d=20/9\approx 2.2$.

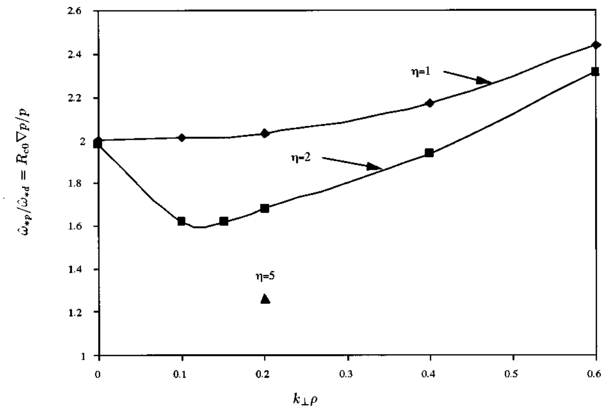

FIG. 1. Critical pressure gradient vs $k_\perp\rho_i$ for $\eta=2/3$ and 2. The $\eta=5$ prediction is also shown.

We consider first the stability of collisionless interchange modes, i.e., modes with $\overline{\phi}=\phi$. We will pay particular attention to the effect on stability of FLR and of profile dependence. Consider first the case with $\eta=1$. In Fig. 1 we display the normalized outer midplane critical pressure gradient $\hat{\omega}_{*p}/\hat{\omega}_{*d}=R_{c0}\nabla p/p$ versus the ion gyro radius $k_\perp\rho_i$. (The collisionless interchange mode is a kinetic version of the MHD interchange mode.) The effect of FLR is observed to be stabilizing, i.e., a larger $k_\perp\rho_i$ value permits a steeper pressure gradient. Since $k_\perp\approx m/R_0$ the most unstable mode has $k_\perp\rho\sim\rho/R_0\ll 1$. The kinetic theory prediction is comparable with the MHD prediction but not identical since it leaves out the $v_\parallel$ part of the curvature drive but takes proper account of the wave particle resonance interaction.

The destabilization that results from increasing $\eta>1$ is an important and new result. Figure 1 compares the stability boundary for both the $\eta=2$ and the $\eta=1$ modes. When $\eta>1$ the Nyquist analysis indicates two unstable roots when the pressure gradient exceeds a critical value. The two modes predicted are the fast growing MHD-like mode and a drift frequency mode.[5] Figure 1 demonstrates that for $\eta=2$ the critical pressure gradient can be limited to values substantially below the limit set by the $\eta=1$ interchange mode. A value for $\eta=5$ is also shown in Fig. 1. The degradation of the stability that is observed as η increases derives from the profile effects that underlie the charge separation. Notice also that the η-driven degradation of stability comes from the FLR coupling terms and disappears when $k_\perp\rho_i=0$.

Stability is also degraded when $\eta<1$ see (Table I), i.e.,

TABLE I. $\hat{\omega}_{*p}/\hat{\omega}_d$ $(=R_{c0}\nabla p/p)$ for collisionless interchange and localized modes.

η	$k_\perp\rho_i$	f_T	$\hat{\omega}_{*p}/\hat{\omega}_d$
0.5	0.2	1	0.39
1	0.2	1	2.03
2	0.2	1	1.68
1	0.2	0.8	2.54

for too strongly peaked density profiles. Thus there is an optimum density and temperature profile for which the charge separation that accompanies an interchange of flux tubes is minimized and a deviation from this profile is destabilizing. This comes about because, whereas for a marginally stable MHD pressure profile an interchange of flux tubes does not change the pressure profile or the internal plasma energy, for a critical η value the same exchange of flux tubes leaves the density and temperature profiles unchanged.

For comparison we recall that when $\omega_* \gg \omega_d$ an interchange mode is unstable for all ω_* when $\omega_d > 0$, i.e., in ''bad curvature.'' The toroidal η_i mode discussed in the η_i section below is closely related to the η-driven interchange mode just discussed.

B. Trapped particle mode

From Eq. (6b) we observe that the curvature drift is largest for the most deeply trapped particles, i.e., particles trapped at the outer midplane where the magnetic field has its minimum value. Therefore one may expect a tendency for curvature driven modes to localize in the vicinity of the outer midplane, which would give rise to ''trapped particle'' modes. To evaluate the tendency of modes to localize at the outer midplane we will assume that the wave is localized near the outer midplane so that a fraction f_T of deeply trapped particles feel the full wave potential, i.e., $\phi = \phi_0$, while the shallowly trapped and circulating particles only feel a small bounce averaged potential, i.e., $\bar{\phi}/\phi_0 \ll 1$. Nyquist studies reveal that trapped particle modes, i.e., modes with $f_T < 1$ are always more stable than interchange modes. For example, for $\eta = 1$, $k_\perp \rho = 0.2$, $f_T = 0.8$, we find that there is instability when $\omega_{*p} > 2.54\ \hat{\omega}_d$ whereas for the same case but with $f_T = 1$ we find instability when $\omega_{*p} > 2.03\ \hat{\omega}_d$ (Table I). This result is peculiar to stabilization by compressibility and indicates that since the deeply trapped particles provide the compressibility, a mode that is felt by more deeply trapped particles is more stable.

C. Collisional electron modes, the dipole η_i mode

Electrons are more collisional than ions at the same temperature and the difference in the electron and ion responses deriving from the difference in collisionality can drive instability. When the electrons are sufficiently collisional they have an adiabatic response to fluctuating electric fields. Modes that result from collisional (adiabatic) electrons and collisionless ions can be viewed as a limiting case of the dissipative trapped ion mode. This mode is destabilized as η_i increases and is analogous to the tokamak toroidal η_i mode.[14]

To obtain a estimate of this instability we assume that the electron response is adiabatic, i.e., $\tilde{f} = q\phi F_{0\epsilon}$. Then the sum over species in Eq. (9) is reduced to the ion term. In Fig. 2 we plot the marginal stability boundary $\hat{\omega}_{*p}/\hat{\omega}_d$ vs $k_\perp \rho_i$ with $\eta_i = 2$ for both the collisional electron mode and for the collisionless interchange mode. We observe that the collisional electron mode does not become unstable unless the pressure gradient is about a factor of 2 larger than the critical gradient for the interchange mode (which is the FLR corrected fast MHD mode). For general values of η_i we find that the collisional electron mode is stable at $\eta_i = 1$ and becomes unstable for sufficiently large $\hat{\omega}_{*p}/\hat{\omega}_d$ when $\eta_i > 1$ but is always stable when the collisionless interchange mode is stable. Unlike the η-driven collisionless interchange mode discussed above this mode is only unstable for $\eta_i > 1$.

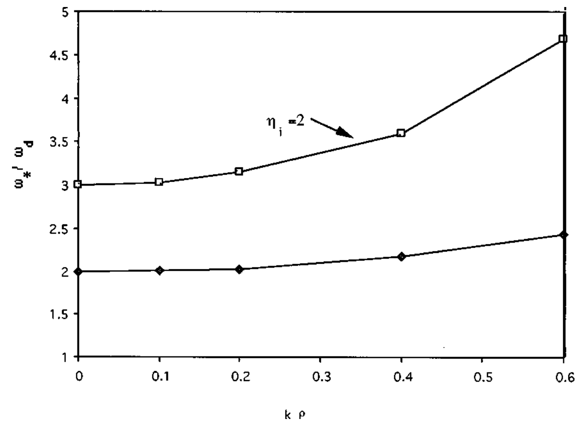

FIG. 2. The marginal stability boundary $\hat{\omega}_{*p}/\hat{\omega}_d$ vs $k_\perp \rho_i$ with $\eta_i = 2$ for both the collisional electron (η_i) mode and for the MHD-like collisionless interchange mode (dashed).

The η_i modes are widely discussed in tokamak literature. The slab-η_i mode[15] has a finite $k_\parallel$ and is driven by coupling to sound waves whereas we are dealing here with a $k_\parallel \sim 0$ mode. On the other hand the toroidal η_i mode[14] has a ballooning character and is driven by the bad curvature curvature that is located on the outside of a tokamak. We have shown that in a dipole this mode appears with an interchange character.

D. Modes that depend on parallel dynamics

Drift modes can depend on parallel dynamics as is the case with the so-called ''universal instability'' and the slab ''η_i mode.'' These modes have finite $k_\parallel$ and do not conserve the second invariant, J, and it is therefore not appropriate to bounce average the drift kinetic equation for the ion response.

To analyze the stability of modes with finite $k_\parallel$ we apply Eq. (2) for the ion response and keep the $v_\parallel$ velocity dependence in the ion curvature drift frequency. Using Eq. (3c) and averaging the perpendicular ion drift we obtain the estimate

$$\omega_{di} \approx \frac{k_\perp}{\Omega_0 R_c} (v_\parallel^2 + T_i). \tag{10}$$

For electrons we assume an adiabatic response since $k_\parallel v_e \gg \omega$. Substituting Eq. (10) into (2) and replacing $v_\parallel \mathbf{b} \cdot \nabla$ by $k_\parallel v_\parallel$ in the ion term we obtain the follow dispersion relation:

$$0 = \frac{2\phi}{T} - \frac{f_T(\phi/T)\omega_{*nj}}{(v_t^2 k_\parallel^2 - 8\omega\omega_{di})^{1/2}} [(\omega/\omega_{*ni} + 1 - \eta_i/2) \times (1 - (k_\perp \rho_i)^2)(W(\zeta_+) - W(\zeta_-)) + \eta_i (1 - (k_\perp \rho_i)^2) \times (\zeta_+ - \zeta_- + \zeta_+^2 W(\zeta_+) - \zeta_-^2 W(\zeta_-))], \tag{11}$$

with

$$\zeta_{\pm}=\frac{k_{\|}R_c\Omega_{ci}}{2k_{\perp}v_t}\pm\left[\left(\frac{k_{\|}R_c\Omega_{ci}}{2k_{\perp}v_t}\right)^2-\frac{\omega}{2\omega_{di}}\right]^{1/2}. \quad (12)$$

The thermal speed $v_t^2=2T_i/m_i$, $\omega_{di}=k_{\perp}T_i/(m_iR_0\Omega_{ci0})$ and

$$W(\zeta)=\frac{1}{\sqrt{\pi}}\int_{-\infty}^{\infty}\frac{e^{-x^2}dx}{x-\zeta}.$$

For $\mathrm{Im}(\zeta)>0$, W is equal to the plasma dispersion function[16] $Z(\zeta)$, and for $\mathrm{Im}(\zeta)<0$, $W(\zeta)=-Z(-\zeta)$.

The Nyquist analysis of Eq. (11) varying η, $k_{\perp}\rho$, and $\hat{\omega}_*/\hat{\omega}_D$ always indicates stability. This is not surprising because it is known that Landau damping will be strongly stabilizing unless $v_i\ll\omega/k_{\|}\ll v_e$. In a levitated dipole the field lines are closed and we can impose the condition $k_{\|}\sim m/R_c$ with R_c the radius of curvature and $m\geq 1$. Since $\omega\sim\omega_d=v_i(k_{\perp}\rho_i)/R_c$ the first inequality $v_i\ll\omega/k_{\|}$ can be written as $k_{\perp}\rho_i\gg 1$. This indicates that the mode will be strongly Landau damped when $k_{\perp}\rho_i<1$. The observed stability of these modes is a result of the field line length being insufficient in a dipole field to support the relatively long parallel wavelength of these modes.

IV. CONCLUSIONS

In a previous study[5] we have shown that a number of electrostatic drift frequency modes become stable when the pressure gradient in the plasma satisfies the MHD interchange stability criterion. In this work we have followed a more general approach to this problem that utilizes the Nyquist criterion to track stability boundaries obtained from a general dispersion relation with a minimum of simplifying assumptions. MHD stability requires that the pressure gradient not exceed a critical value $R_{c0}\nabla p/p<\gamma$ and we find that while the kinetic theory predicts a similar result for interchange modes the profile and FLR effects can significantly alter stability.

We have shown that a confinement scheme that is marginally stable due to the balance of compressibility and curvature drive possesses unusually good stability properties. We find that when $\eta=1$ and $\omega_{*p}<2\omega_d$ all of the studied interchange modes, η-driven modes, localized (trapped particle) modes, η_i (i.e., collisional electron modes) and modes that depend on parallel dynamics were observed to be stable. In the regime that is relevant to present day experiments the dipole η_i mode is destabilized when $\eta_i>1$ but it is seen to be stable when the MHD stability criterion is satisfied. In the collisionless reactor regime, when $\eta\neq 1$ ($\eta_i=\eta_e=\eta$) the η-driven interchange mode can be unstable and therefore one must either maintain $\eta\sim 1$ or accept the consequences of the instability. The nonlinear consequence of this instability may be the development of convective cells which could serve to maintain a critical density profile.

For the collisionless curvature driven modes we have seen that extended, i.e., interchange modes, are more unstable than modes that are localized in the low field region. This result is opposite to the standard trapped particle result and may be understood as an indication that the deeply trapped particles are localized to the region with the largest field gradient and therefore have the largest compressibility driven stabilization.

It is believed that the η_i mode plays an important role in thermal transport for a tokamak. The η_i mode is driven by the temperature gradient and there is an associated pressure gradient. For a tokamak this mode can be unstable while the plasma maintains MHD stability due to the good curvature in the outer torus. For a dipole the pressure gradient associated with the temperature gradient will destabilize interchange modes before it will destabilize η_i modes. Stabilization from compressibility requires that $R_{c0}\nabla p/p<\gamma$ ($\gamma=5/3$). For a tokamak $\langle R_c\rangle\nabla p/p\approx A$, with A the aspect ratio and since a tokamak will typically have $A\gtrsim 2$ compressibility will not normally be important. A low aspect ratio tokamak, on the other hand, can gain substantial stabilization from compressibility.

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