Effects of Hot Electrons on Plasma Stability in Closed Magnetic Field Line Geometry*

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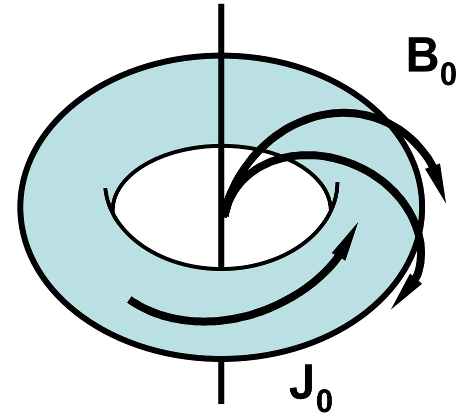
Abstract

Motivated by the electron cyclotron heating being employed on dipole experiments, the effects of a hot species on stability in closed magnetic field line geometry are investigated. The interchange stability of a plasma of background electrons and ions with a fraction of hot electrons is considered. The species diamagnetic drift and magnetic drift frequencies are assumed to be of the same order, and the wave frequency is assumed to be much larger than the background drift frequencies. The background plasma is treated as a single fluid, while a fully kinetic description is employed for the hot species. It is found that geometrical effects significantly complicate the analysis. In general dipolar geometry, poloidal variations of electric and magnetic fields cause the dispersion relation to become an integro-differential equation, which without approximations can only be solved numerically. To examine the possibility of at least a partially analytic solution as well as to obtain an intuitive understanding of instabilities we examine a point dipole and consider the effects of hot electrons to be small and introduce them pertubatively. The dispersion relation is analyzed for the frequency range much smaller as well as of the same order as the hot electron magnetic drift frequency. Two regimes of pressure balance are examined: one dominated by hot electrons and another with the background and hot pressures being comparable.

Plasma Model



- Dipole geometry

$$\vec{\mathbf{B}}_0 = \nabla\psi \times \nabla\zeta; \quad \vec{\mathbf{J}}_0 = R^2 \frac{dp_0}{d\psi} \nabla\zeta$$

- Plasma
  - Background fluid electrons, n_e, T_e
  - Background fluid ions, n_i, T_i
  - Kinetic hot electrons, $n_h << n_e, n_i$, $T_h >> T_e$, T_i
- Motivation
  - Interchange stability of ECH heated electrons

Background plasma

- Equilibrium : $$\nabla \cdot \left(\frac{\nabla \psi}{R^2} \right) + \mu_0 \frac{dp_0}{d\psi} = 0$$

- Perturbations : $$\mathbf{C}_1 = \hat{\mathbf{C}}_1(\psi, \theta) e^{-i\omega t + il\zeta}$$

- First order: $$-m_i n_{0i} \omega^2 \vec{\boldsymbol{\xi}} = e n_{0h} \vec{\mathbf{E}}_1 + \vec{\mathbf{J}}_{0b} \times \vec{\mathbf{B}}_1 + \vec{\mathbf{J}}_{1b} \times \vec{\mathbf{B}}_0 - \nabla p_{1b}$$

$$\vec{\mathbf{E}}_1 = i\omega \vec{\boldsymbol{\xi}} \times \vec{\mathbf{B}}_0 \qquad \vec{\mathbf{B}}_1 = \nabla \times \left(\vec{\boldsymbol{\xi}} \times \vec{\mathbf{B}}_0 \right) \qquad \vec{\mathbf{J}}_1 = \tfrac{1}{\mu_0} \nabla \times \vec{\mathbf{B}}_1$$

$$p_{1b} = -\gamma p_{0b} \nabla \cdot \vec{\boldsymbol{\xi}} - \vec{\boldsymbol{\xi}} \cdot \nabla p_{0b}$$

$$-\frac{\vec{\mathbf{v}}_1}{i\omega} = \vec{\boldsymbol{\xi}} = \xi_B \frac{\vec{\mathbf{B}}_0}{B_0^2} + \xi_\psi \frac{\nabla \psi}{|\nabla \psi|^2} + \xi_\zeta \frac{\nabla \zeta}{|\nabla \zeta|^2}$$

with $n_{0i} = n_{0e} + n_{0h}$, $\gamma = 5/3$

Background plasma

- Quasi-neutrality:

$$n_{1h} = n_{1i} - n_{1e} = \tfrac{1}{i\omega e} \nabla \cdot \vec{\mathbf{J}}_{1b}$$

- $\nabla \psi$ Component of Ampere's Law:

$$ilQ_B - \vec{\mathbf{B}}_0 \cdot \nabla\left(R^2 Q_\zeta\right) = \mu_0 \vec{\mathbf{J}}_{1b} \cdot \nabla \psi + \mu_0 \vec{\mathbf{J}}_{1h} \cdot \nabla \psi$$

with $\vec{\mathbf{B}}_1 = Q_B \frac{\vec{\mathbf{B}}_0}{B_0^2} + Q_\psi \frac{\nabla \psi}{|\nabla \psi|^2} + Q_\zeta \frac{\nabla \zeta}{|\nabla \zeta|^2}$

Hot electrons

- Hot electrons $\Omega_e \geq \omega_b \sim \vec{\mathbf{v}}_{\|} \cdot \nabla >> \omega_{dh} \sim \omega_{*h} >> \omega$
- First order:

$$f_{1h} \approx f_{Mh} \left\{ \frac{e\Phi}{T_h} - \frac{\left(\omega - \omega_{*h}^T\right)}{\left(\omega - \langle \omega_D \rangle_\tau\right)} \left[\frac{e\Phi}{T_h} - \frac{m\mathrm{v}^2 Q_B}{2T_h \overline{B}^2} \frac{\lambda \oint (\overline{B}/B_0) d\tau}{\oint d\tau} \right] \left(1 + \frac{ilm\vec{\mathbf{v}}_\perp \cdot \nabla\psi}{eR^2 B_0^2} \right) \right\}$$

where $\omega_{*h}^T = -\frac{lT_h}{e} \frac{d\ln n_{0h}}{d\psi} \left[1 + \frac{d\ln T_h}{d\ln n_{0h}} \left(\frac{m\mathrm{v}^2}{2T_h} - \frac{3}{2} \right) \right],$

$$\langle \omega_D \rangle_\tau = \frac{\oint \omega_D d\tau}{\oint d\tau} = \frac{lm\mathrm{v}^2}{e} \oint \frac{\vec{\boldsymbol{\kappa}} \cdot \nabla\psi}{R^2 B_0^2} \left[1 - \frac{B_0(1+s)}{2\overline{B}} \lambda \right] d\tau / \oint d\tau$$

with $s = 1 - \frac{\nabla\psi \cdot \nabla \ln B_0}{\vec{\boldsymbol{\kappa}} \cdot \nabla\psi}, \quad \vec{\boldsymbol{\kappa}} = (\hat{\mathbf{b}} \cdot \nabla)\hat{\mathbf{b}}, \quad \hat{\mathbf{b}} = \frac{\vec{\mathbf{B}}_0}{B_0}, \quad \lambda = \frac{\mathrm{v}_\perp^2}{\mathrm{v}^2} \frac{\overline{B}}{B_0}.$

time along trajectory $d\tau \equiv \frac{d\theta}{\vec{\mathbf{v}}_{\|} \cdot \nabla\theta} > 0$

Hot electrons

- Assumptions:
  - High mode number $\Rightarrow\ l >> 1$
  - Coulomb gauge $\Rightarrow\ A_\zeta / \left(A_\psi \sim A_{\|}\right) \sim 1/l << 1$
  - Interchange mode $\Rightarrow\ Q_\psi = \vec{\mathbf{B}}_0 \cdot \nabla \xi_\psi = 0$
    - $\mathbf{\Phi}$ is up-down symmetric, and flux function
    - $A_{\|}$ is up-down asymmetric $\Rightarrow\ \oint \mathrm{v}_{\|} A_{\|} d\tau = 0$
    - $\vec{\mathbf{J}}_{1h} \cdot \vec{\mathbf{B}}_0 \propto \oint \mathrm{v}_{\|} \bar{g}_1 d\vec{\mathbf{v}} = 0$
  - Near marginality $\vec{\mathbf{J}}_{1h} \cdot \nabla \psi,\ Q_B,\ \nabla \cdot \vec{\xi}$ are flux functions
- $$\frac{n_{1h}}{n_{0h}} = \frac{e\langle \Phi \rangle}{T_h} \langle G \rangle + \frac{\langle Q_B \rangle}{\langle B_0^2 \rangle} \langle H \rangle; \quad \frac{\mu_0 \vec{\mathbf{J}}_{1h} \cdot \nabla \psi}{ilB_0^2} = \frac{e\langle \Phi \rangle}{T_h} \frac{\langle \beta_h \rangle}{2} \langle F \rangle - \frac{\langle Q_B \rangle}{\langle B_0^2 \rangle} \frac{\langle \beta_h \rangle}{2} \langle I \rangle$$

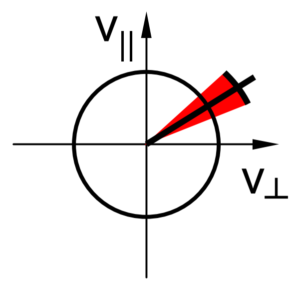
with $\langle \ldots \rangle = V^{-1} \oint (\ldots) d\theta / \vec{\mathbf{B}}_0 \cdot \nabla \theta; \quad V = \oint d\theta / \vec{\mathbf{B}}_0 \cdot \nabla \theta$

Dispersion relation

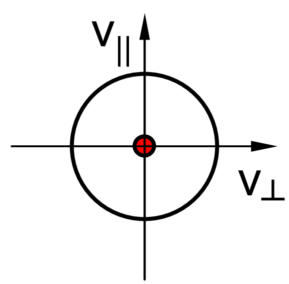
$$A\frac{\omega^2}{\langle\omega_{de}\rangle^2} + B\frac{\omega}{\langle\omega_{de}\rangle} + C = 0$$

where $\langle\omega_{de}\rangle e / lT_e = -d\ln V / d\psi$

- Resonant effects
  - Strong resonance: $\langle\omega_D\rangle_\tau$ reverses sign at $\lambda_{crit} = 2\overline{B}/B_0(1+s)$ happens only when $s > 1$ coefficients are complex



  - Weak resonance: $\langle\omega_D\rangle_\tau \sim \omega$ happens only for $\mathrm{v} \to 0$, coefficients depend on ω



Dispersion relation

- Ignoring resonant effects – quadratic equation.
- Ordering issues:
  - $\beta_b << \beta_h \sim 1$ always stable, since $d < \gamma$

$$\left(1+\frac{\langle\beta_h\rangle}{2}\frac{\langle I\rangle}{\langle B_0^2\rangle\langle B_0^{-2}\rangle}\right)\left[\frac{\omega^2}{\langle\omega_{de}\rangle^2}\langle b\rangle-\frac{n_{0h}T_e}{p_{0b}}\frac{\omega}{\langle\omega_{de}\rangle}\left(1+\frac{d\ln n_{0h}}{d\ln V}\right)-(\gamma-d)\right]=0$$

  - $\beta_h \sim \beta \sim 1 \Rightarrow n_{0h}T_e / p_{0b} \sim T_e / T_h << \langle\omega_{de}\rangle / \omega$

$$\frac{\omega^2}{\langle\omega_{de}\rangle^2}=\frac{(\gamma-d)\left(1+\frac{1}{2}d\langle\beta_b\rangle+\frac{\langle\beta_h\rangle}{2}\langle I\rangle/\langle B_0^2\rangle\langle B_0^{-2}\rangle\right)}{\langle b\rangle\left(1+\frac{1}{2}\gamma\langle\beta_b\rangle+\frac{\langle\beta_h\rangle}{2}\langle I\rangle/\langle B_0^2\rangle\langle B_0^{-2}\rangle\right)}$$

  - $\beta_b << \beta_h \Rightarrow n_{0h}T_e / p_{0b} \sim \langle\omega_{de}\rangle / \omega >> T_e / T_h$

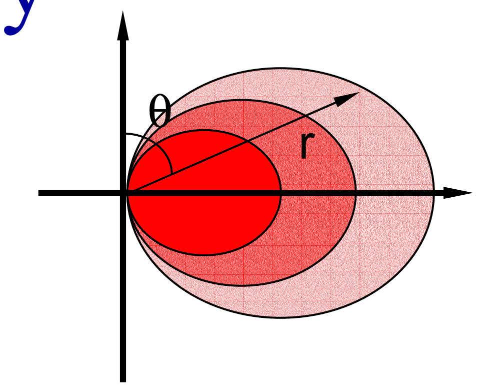
where $d=-\frac{d\ln p_{0b}}{d\ln V}$, $\beta_b=\frac{2\mu_0 p_{0b}}{B_0^2}$, $\beta_h=\frac{2\mu_0 p_{0h}}{B_0^2}$, $b=\frac{l^2 m_i n_{0i} T_e^2}{e^2 p_{0b} B_0^2 R^2}$

Dispersion relation

- Resonant effects
  - Strong resonance – always unstable (for our f_{0h})
  - Weak resonance – let $\omega = \omega_0 + \omega_1$
    - $\beta_b << \beta_h \sim 1, \ \dfrac{\omega_1}{\omega_0} \propto i\omega_{*h}\left(1 - \frac{3}{2}\eta_h\right)$

      stable for $\dfrac{d \ln n_{0h}}{d\psi} \geq \dfrac{3}{2}\dfrac{d \ln T_h}{d\psi}$
    - Other cases have to be solved numerically

Point dipole geometry



- Separable solution:

$$\psi(r,\mu)=\psi_0 h(\mu)\left(\frac{r_0}{r}\right)^{\alpha}$$

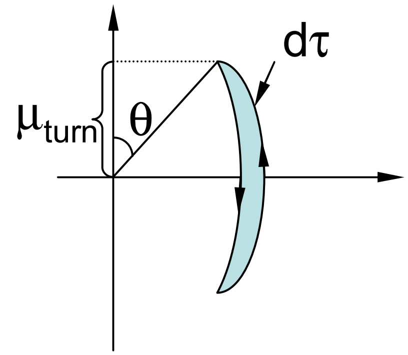
where $\mu=\cos\theta$

- – Grad-Shafranov: $\frac{d^2h}{d\mu^2}=-\frac{\alpha(\alpha+1)}{(1-\mu^2)}h-\alpha(\alpha+2)\beta_0 h^{1+4/\alpha}$

  where $\beta_0=\frac{2\mu_0 p_0 r_0^4}{\alpha^2\psi_0^2}$, $p(\psi)=p_0\left(\frac{\psi}{\psi_0}\right)^{2+4/\alpha}$
- – For given β_0 solve for $h(\mu),\alpha$ then express all equilibrium quantities in terms of them, e.g.

$$\vec{\mathrm{B}}_0=\frac{\psi_0}{r_0^2}\left(\frac{r_0}{r}\right)^{\alpha+2}\left(-\frac{\partial h}{\partial\mu}\hat{\mathbf{r}}+\frac{\alpha h}{\sqrt{1-\mu^2}}\hat{\boldsymbol{\theta}}\right),\quad \frac{d\theta}{\vec{\mathbf{B}}_0\cdot\nabla\theta}=d\mu\frac{r_0^3\psi_0^{3/\alpha}}{\alpha\psi^{1+3/\alpha}}h^{3/\alpha}$$

Point dipole geometry



- Drift reversal – sign of $\langle \omega_D \rangle_\tau = \frac{\oint \omega_D d\tau}{\oint d\tau}$

  where trajectory average is defined by

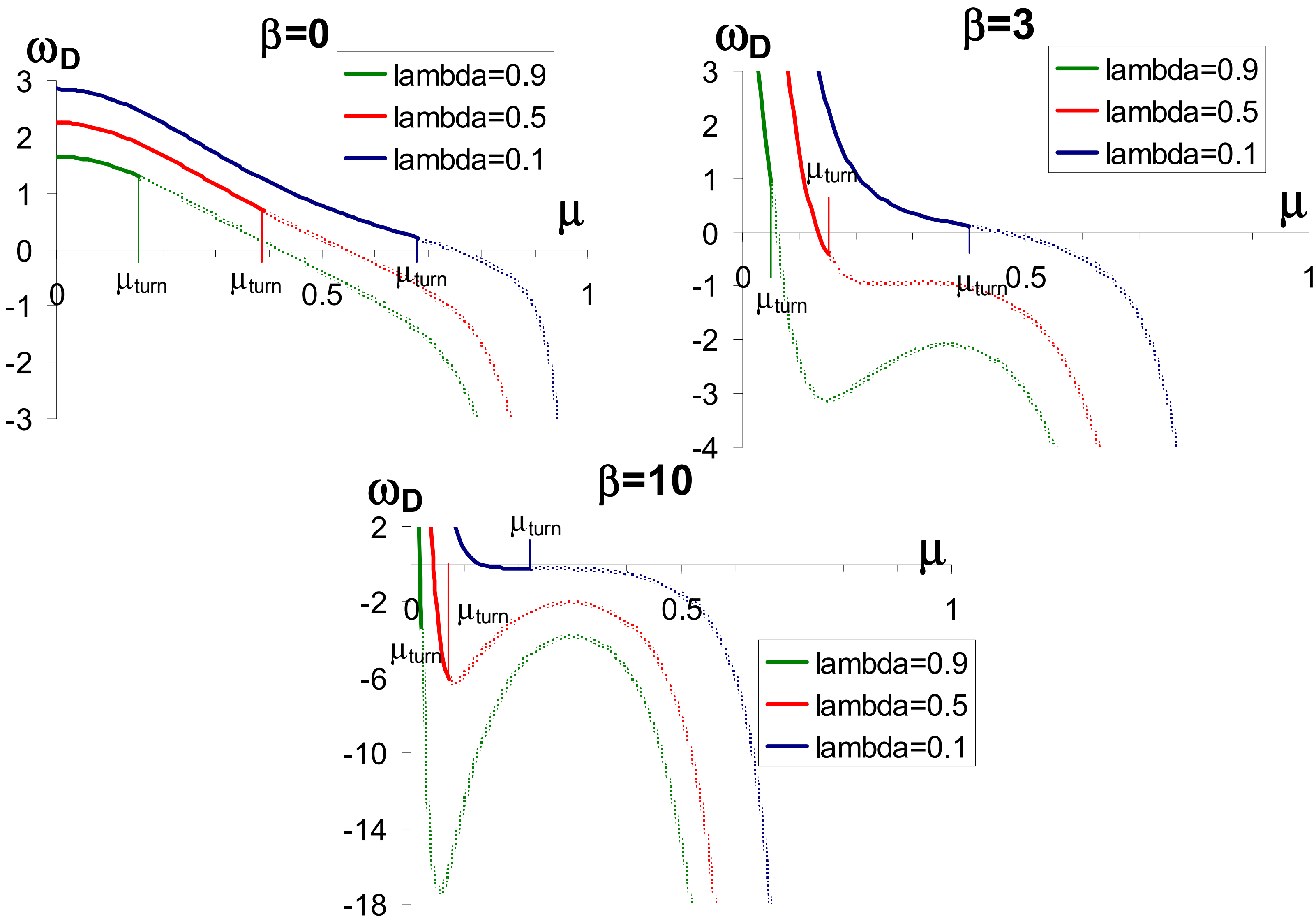
$$\frac{\oint (\ldots) d\tau}{\oint d\tau} = \frac{\oint d\mu (\ldots) h^{1/\alpha - 1} \sqrt{\left(\frac{dh}{d\mu}\right)^2 + \frac{\alpha^2 h^2}{1-\mu^2}} \Big/ \sqrt{1 - \frac{\lambda}{\alpha}\sqrt{\left(\frac{dh}{d\mu}\right)^2 + \frac{\alpha^2 h^2}{1-\mu^2}}}}{\oint d\mu h^{1/\alpha - 1} \sqrt{\left(\frac{dh}{d\mu}\right)^2 + \frac{\alpha^2 h^2}{1-\mu^2}} \Big/ \sqrt{1 - \frac{\lambda}{\alpha}\sqrt{\left(\frac{dh}{d\mu}\right)^2 + \frac{\alpha^2 h^2}{1-\mu^2}}}}$$

  with the turning points given by

$$\frac{\alpha^2}{\lambda^2} = \left(\frac{dh(\mu_{turn})}{d\mu}\right)^2 + \frac{\alpha^2 h^2(\mu_{turn})}{1-\mu_{trun}^2}$$

- Drift reversal does not occur.

Point dipole geometry



Point dipole geometry

- Stability analysis without resonant effects
  - $\langle I \rangle > 0$
    - $\beta_b \sim \beta_h \sim 1$ and $\beta_b << \beta_h \sim 1$ are always stable since $0 \le d \le \gamma$
- Stability analysis with resonant effects
  - $\beta_b \sim \beta_h$
    Current calculations suggest that stability requirement is identical to $\beta_b << \beta_h \sim 1$ e.g. $\beta_b = \beta_h = 3$
  - Need to evaluate more cases – work in progress

Conclusions

- Analysis of hot electron effects on the interchange stability of dipolar plasma is similar to Z-pinch. Geometrical complications force the problem to become purely computational
- Semi-analytical solution is possible for point dipole approximation.
  - Drift reversal does not occur, so only weak resonance can make plasma unstable.
  - For $\beta_b << \beta_h \sim 1,\ \beta_b \sim \beta_h$ plasma is stable to interchange modes in the absence of resonant effects. Interaction of resonant electrons with the wave require $d \ln n_{0h} / d\psi \geq \frac{3}{2} d \ln T_h / d\psi$ for stability
  - Further calculations are required for $\beta_b << \beta_h$ case in point dipole geometry.