Comparison of experimental measurements and gyrokinetic turbulent electron transport models in Alcator C-Mod plasmas

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- TRANSP, GYRO and synthetic PCI
- Turbulence
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Introduction

- The experiments were carried out over the range of densities covering the "neo-Alcator" ($\tau_e \propto \bar{n}_e$) to the "saturated ohmic" regime.

- Low density ohmic plasma in the neo-Alcator regime, where the thermal energy is mainly lost through the electron channel, is an excellent candidate to unambiguously investigate electron transport.
Approaches

**Turbulence**
- Synthetic Diagnostic
- $\tilde{n}_e$
- $\int \tilde{n}_e \, dl$
- Phase Contrast Imaging

**Thermal Transport**
- $\chi_{e,i,eff}$
- $\chi_{e,i,eff}$
- TRANSP

**GYRO**
A phase contrast imaging (PCI) diagnostic is used to measure turbulence.

- PCI measures density fluctuations along 32 vertical chords.

  | Wavenumber Range: |
  | $0.5\text{cm}^{-1}<|k_R|< 55 \text{cm}^{-1}$ |
  | Frequency Range: |
  | $2\text{kHz} \sim 5\text{MHz}$ |

- The localization upgrade allows PCI to resolve the direction of propagation of the measured turbulence in ITG and TEM range.

- The improved calibration also allows for the intensity of the observed fluctuations to be determined absolutely.

- It is almost impossible to directly infer $\tilde{n}/n$ from $\int \tilde{n}_e \, dl$. 
GYRO* is used to predict fluctuations and associated transport.

- **GYRO is an initial value Eulerian (continuum) 5D gyrokinetic code**

- **GYRO is well-documented and user-friendly.**

- **GYRO contains the following ingredients for quantitatively accurate transport predictions**
  - takes measured experimental profiles as inputs
  - local and global
  - realistic geometry (Miller formulation)
  - trapped and passing electrons
  - finite beta (magnetic fluctuations)
  - equilibrium sheared ExB

*http://fusion.gat.com/theory/Gyro*
TRANSP is used to calculate thermal diffusivities $\chi$

- **TRANSP calculates thermal diffusivities from energy balance equation**

\[
\frac{3}{2} n_j \frac{dT_j}{dt} = -\nabla \cdot q_j + Q_j - P_j : \nabla u_j + S_E
\]

\[
\chi_j = \frac{q_j}{n_j \nabla T_j} = \frac{\int_0^r dV \nabla \cdot q_j}{A_{\text{surface}} n_j \nabla T_j} = \frac{P_{\text{cond}j}}{A_{\text{surface}} n_j \nabla T_j}
\]

\[
\chi_{\text{eff}} = \frac{n_e \chi_e \nabla T_e + n_i \chi_i \nabla T_i}{n_e \nabla T_e + n_i \nabla T_i}
\]

$q_j$ : heat flux

$Q_j$ : energy exchange due to collisions

$P_j$ : pressure tensor

$u_j$ : fluid velocity

$S_E$ : external source of energy
A synthetic PCI diagnostic is used for quantitative code-experiment comparisons

- A synthetic PCI diagnostic has been developed to post-analyze GYRO output and emulate the PCI instrumental response.
- The same data analysis package is used to process the experimental and emulated PCI signals.

C. Rost, to be submitted (2009).
Turbulence

GYRO $\tilde{n}_e$ Synthetic Diagnostic $\int \tilde{n}_e dl$ Phase Contrast Imaging
PCI has established the fluctuations above ~80 kHz are dominated by the mode propagating in the ion diamagnetic direction and from $r/a < 0.85$

Available Er measurements indicate that the background Doppler rotation is not enough to reverse the mode propagating direction of the turbulence with the wavenumbers in the ITG and TEM regime.

Saturated ohmic $\overline{n}_e = 0.93 \times 10^{20} \text{m}^{-3}$
PCI has established the fluctuations above \(\sim 80 \text{ kHz}\) are dominated by the mode propagating in the ion diamagnetic direction and from \(r/a<0.85\)

Localization depends on the magnitude of the magnetic pitch angle \(\zeta\)

Saturated ohmic \(\bar{n}_e = 0.93 \times 10^{20} \text{ m}^{-3}\)

- Available Er measurements indicate that the background Doppler rotation is not enough to reverse the mode propagating direction of the turbulence with the wavenumbers in the ITG and TEM regime.
Nonlinear GYRO simulation shows that the ITG mode is the dominant turbulence.

A separate global simulation shows no significant density fluctuation ($\bar{n}_e/n_e<0.3\%$) in $r/a<0.35$. 

$f<0$, $i^+$ direction

$f>0$, $e^-$ direction.

Saturated ohmic $\bar{n}_e = 0.93 \times 10^{20} \text{ m}^{-3}$
Assuming $v_{E\times B} \sim 2 \text{ km/sec}$ for the Doppler shift, the synthetic and experimental spectra are similar.

- Simulations cover $0 \leq k_{\theta}\rho_s \leq 1.5$ and $0.4 \leq r/a \leq 0.8$ with $m_i/m_e = 3600$.

- Turbulence below 80 kHz might be localized at the plasma edge, where the turbulence propagating in the ion and electron diamagnetic directions may be comparable.
The wavenumber spectrum of density fluctuations in the frequency range of 80-250 kHz quantitatively agrees with GYRO simulation. Saturated ohmic $\bar{n}_e = 0.93 \times 10^{20} \text{m}^{-3}$

- The uncertainties are mainly from the absolute calibration ~50%.
Density fluctuation intensities in the 80-250 kHz frequency range quantitatively agree with GYRO for a range of densities.
In the linear ohmic regime, $\chi_i$ is much smaller than $\chi_e$.

- As $n_e$ increases, $\chi_e$ decreases, but $\chi_i$ increases slowly, $\chi_{\text{eff}}$ also decreases.
- In the saturated ohmic regime, $\chi_i$ becomes comparable to $\chi_e$.
Agreement between experiment and theory in $\chi_{\text{eff}}$ is obtained after reducing $a/L_{\text{Ti}}$ by 20%

- Simulation covers

$$0.0 \lesssim k_{\phi} \rho_s \lesssim 1.5$$

with

$$m_i / m_e = 3600$$

- Diffusivities are averaged over

$$0.4 \lesssim r/a \lesssim 0.8$$

- The estimated uncertainty of $a/L_{\text{Ti}}$ is as much as 30% in ohmic plasmas

- $\varepsilon$ is the reduction factor of $a/L_{\text{Ti}}$

$$\left( \frac{a}{T_i} \frac{\partial T_i}{\partial r} \right)_{\text{sim}} = (1 - \varepsilon) \frac{a}{T_i} \frac{\partial T_i}{\partial r}$$

- Simulated fluctuation intensities still agree with experiments within the experimental uncertainty after the $a/L_{\text{Ti}}$ reduction
At higher densities, agreement between experiment and code predictions of $\chi_e$ and $\chi_i$ is obtained after the $a/L_{Ti}$ reduction; however, at the lowest density, the experimental values of $\chi_e$ and $\chi_i$ disagree with code results.

- $\varepsilon$ is the reduction factor of $a/L_{Ti}$:
  \[
  \left( \frac{a}{T_i} \frac{\partial T_i}{\partial r} \right)_{sim} = (1 - \varepsilon) \left( \frac{a}{T_i} \frac{\partial T_i}{\partial r} \right)
  \]
To further explore the discrepancy in thermal diffusivity between experiments and simulations in the linear ohmic regime, we have investigated the impact of

- ExB shear suppression
- trapped electron mode
- collisionality
- finite-\(\beta\)
- high-k turbulence
The simulated $\chi_i$ can be reduced to the experimental level by further reducing $a/L_{Ti}$ and/or adding $E \times B$ shear, but by doing so the simulated $\chi_e$ is further reduced below the experimental level.

- $\varepsilon$ is the reduction factor of $a/L_{Ti}$: $\left( \frac{a}{T_i} \frac{\partial T_i}{\partial r} \right)_{sim} = (1 - \varepsilon) \frac{a}{T_i} \frac{\partial T_i}{\partial r}$
Significant transport contribution from the TEM turbulence is not likely for the measured temperature and density profiles.

![Electron Thermal Diffusivity](image1)

![Ion Thermal Diffusivity](image2)

- $\varepsilon$ is the reduction factor of $a/L_{Ti}$:
  \[
  \frac{a}{T_i} \frac{\partial T_i}{\partial r} \bigg|_{\text{sim}} = (1 - \varepsilon) \frac{a}{T_i} \frac{\partial T_i}{\partial r}
  \]

- The simulated $\chi_e$ can only be raised to the experimental level after increasing $a/L_{ne}$ by a factor of two where the TEM turbulence becomes significant.
Nonlinear flux-tube simulations show that the impact of varying $v_{ei}$ on turbulent transport is very weak.

<table>
<thead>
<tr>
<th>$v_{ei}/(c_s/a)$</th>
<th>$\chi_e$ [m$^2$/sec]</th>
<th>$\chi_i$ [m$^2$/sec]</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.01</td>
<td>0.67</td>
<td>2.97</td>
</tr>
<tr>
<td>0.05</td>
<td>0.56</td>
<td>2.67</td>
</tr>
<tr>
<td>0.10</td>
<td>0.48</td>
<td>2.32</td>
</tr>
</tbody>
</table>

- $v_{ei}/(c_s/a)=0.05$ corresponds to the experimental measurement.
- The other plasma parameters are taken from $r/a=0.63$ at the lowest density plasma with $\bar{n}_e = 0.34 \times 10^{20}$ m$^{-3}$.
GYRO shows that the contribution from the electromagnetic fluctuations are negligible in the low-β C-Mod Plasmas.

<table>
<thead>
<tr>
<th>β</th>
<th>$\chi_e$ [m$^2$/sec]</th>
<th>$\chi_i$ [m$^2$/sec]</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.00%</td>
<td>0.560</td>
<td>2.670</td>
</tr>
<tr>
<td>0.07%</td>
<td>0.564</td>
<td>2.671</td>
</tr>
<tr>
<td>0.14%</td>
<td>0.577</td>
<td>2.673</td>
</tr>
</tbody>
</table>

- β=0.07% corresponds to the experimental measurement.

- Electromagnetic fluctuations contribute less than 3% of the total simulated electron thermal transport even after doubling the experimental measured β.

- The other plasma parameters are taken from r/a=0.63 at the lowest density plasma with $\bar{n}_e = 0.34 \times 10^{20}$ m$^{-3}$. 
At the lowest density, inclusion of high-k turbulence in the ETG range $k_0\rho_s = 2-8$ accounts for less than 5% of total simulated electron transport; it is not known if including even higher values of $k_0\rho_s$ would significantly change this result.

- After adding $E \times B$ shear suppression and/or reducing $\nabla Ti$, to match $\chi_i$, the electron transport from high-k turbulence is not significantly affected.

- Measurements to date by PCI indicate very low levels of high-k turbulence, falling into the background noise level.

Linear ohmic $\overline{n}_e = 0.35 \times 10^{20} \text{ m}^{-3}$

Electron Energy Diffusion

<table>
<thead>
<tr>
<th>$k_0\rho_s$</th>
<th>Contribution per mode</th>
</tr>
</thead>
<tbody>
<tr>
<td>$k_0\rho_s &lt; 1$</td>
<td>95.2%</td>
</tr>
<tr>
<td>$k_0\rho_s &gt; 1$</td>
<td>4.8%</td>
</tr>
<tr>
<td>$k_0\rho_s &gt; 2$</td>
<td>3.1%</td>
</tr>
</tbody>
</table>

$\times 20$

[Graph showing electron energy diffusion with contributions from different $k_0\rho_s$ ranges.]
Ohmic Drift Velocity

One piece of physics that is omitted in gyrokinetic codes is the potential importance of electron drift velocity:

\[ u_{e\parallel} = \frac{j}{ne} \]
The electron drift velocity is up to 6 times the ion acoustic speed in the linear ohmic regime.

- A correlation exists between the energy confinement time and the ohmic drift velocity.
  \[ \tau_E \propto n q R^2 a \kappa^{0.5} \]

(Goldston, PPCF, 1984)
The electron thermal diffusivity from the TRANSP analysis shows a dependence on $u_{e||}/c_s$. 

\[ \chi_e [m^2/sec] \]

\[ u_{e||}/c_s \]

- 5.2 T; 0.8 MA: ♦
- 5.2 T; 0.6 MA: △
- 5.2 T; 0.4 MA: □
Summary

• The key role played by the ITG turbulence in the saturated ohmic and H-mode plasmas has been verified.
  - Propagation of turbulence is found to be dominantly in the ion diamagnetic direction.
  - Intensity of the ITG turbulence increases with density, in agreement between simulation and experiments.
  - The absolute fluctuation intensity agrees with simulation within experimental error.
  - Simulated $\chi_{\text{eff}}$, $\chi_e$, and $\chi_i$ agree with experiments after varying $a/L_{Ti}$ within 20%.

• At the low densities in the linear ohmic regime, where the electron transport dominates ($\chi_e \gg \chi_i$), GYRO shows $\chi_i > \chi_e$.
  - TEM is unlikely to be important for the measured density and temperature profiles.
  - Inclusion of high-k ($k_0\rho_s \leq 8$) turbulence does not raise $\chi_e$ to the experimental level.
  - GYRO shows that the contributions from the electromagnetic fluctuations are negligible in the low-$\beta$ C-Mod Plasmas.

• Electron drift velocity ($u_{e||}/c_s \geq 1$) associated with the ohmic toroidal plasma current may play an important role.

• Future GYRO and TGLF work will look into turbulent exchange (Waltz-Stabler, PoP, 2008) as another possible remedy.
Appendix
Linear stability analysis in the $a/L_{Ti}$-$a/L_{Te}$ plane shows that ITG remains the most unstable mode even after varying $a/L_{Ti}$ and $a/L_{Te}$ by 50%.

- The other plasma parameters are taken from $r/a=0.63$ at the lowest density plasma with $\bar{n}_e = 0.34 \times 10^{20}$ m$^{-3}$. 
Linear stability analysis in the $a/L_{\text{Ti}}$-$v_{ei}$ plane shows that ITG remains the most unstable mode even after varying $a/L_{\text{Ti}}$ and $v_{ei}$ by 50%.

• Nonlinear flux-tube simulation shows $\chi_e^{\text{sim}} = 0.08 \text{ m}^2/\text{sec}$, which is significantly below the experimental level $\chi_e^{\text{exp}} = 1.5 \pm 0.5 \text{ m}^2/\text{sec}$

• The other plasma parameters are taken from $r/a=0.63$ at the lowest density plasma with $\bar{n}_e = 0.34 \times 10^{20} \text{ m}^{-3}$.