Full wave/Fokker-Planck analysis of driven current and hard X-ray emission profiles during lower hybrid experiments on Alcator C-Mod

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Motivation

- Lower hybrid generates efficient current, only option in outer plasma.
- Assess full wave effects – the computational resources needed to do this now exist.
- Electric field is needed for
  - Direct evaluation of the wave induced quasilinear diffusion accounting for phase interference,
  - rf-sheath interactions at the wall,
  - coupling to Newtonian and Monte-Carlo calculations of plasma response (diffusion, distribution fn evolution).
- Implementation of boundary conditions is well defined.
Conductivity Relation - LHRF

- LHCD Regime: $\Omega_{ci}^2 \ll \omega^2 \ll \Omega_{ce}^2$ and $\omega \geq 2\omega_{LH}; \omega_{LH} = (\Omega_{ci} \Omega_{ce})^{1/2}$
- Unmagnetized ions
- Strongly magnetized electrons $[(k_{\perp} \rho_e)^2 \ll 1]$
- Wave equation is sixth order with two propagating modes, one damped:
- Mode converted ion plasma wave is not propagative, so drop sixth order term
- Electrostatic LH “slow wave” branch
- Electromagnetic LH “fast wave” branch

$$P_4 n_{\perp}^4 + P_2 n_{\perp}^2 + P_0 = 0$$

Wave lengths are very short:

$$\lambda_{\perp} \approx (\omega/\omega_{pe}) \lambda_{\parallel} \approx 1\text{mm}$$

Predicts an accessibility criterion:

$$n_{\parallel} > n_a \equiv \frac{\omega_{pe}}{\Omega_{ce}} + S^{1/2}$$
The Spectral Gap between launched and damped $n_{||}$

- The launched waves will not damp until the phase velocity slows down to about $3v_{te}$.
- Full wave solutions include diffraction effects for this upshift of $n_{||}$.
LH Absorption physics

- Parallel refractive indexes are geometrically up-shifted as waves propagate to smaller major radius. Poloidal asymmetries can cause spread in m spectrum.

\[ n_\parallel = \frac{c}{\omega} \left( \frac{m}{q} + n \right) / R \]

- Quasilinear damping occurs at \( \omega / (k_\parallel v_{te}) \sim 3 \Rightarrow \)

\[ n_\parallel \approx \frac{5.7}{\sqrt{T_e[keV]}} \]

This also sets poloidal resolution.

so lower temperatures require higher \( n_\parallel \) for damping.

- Higher parallel refractive indexes are more accessible to the interior of the plasma but also damp at lower \( T_e \) =larger radii.

- Current drive scales as \( 1/n_{e0} n_\parallel^2 \) and \( n_{acc} [n_{e0},B] \) sets minimum \( n_\parallel \)

\[ \Rightarrow \text{operation in weak damping regime for } T_{e0} < 16 \text{keV} \]
Approaches to solution

- WKB expansions: ray tracing (GENRAY, ACCOME) and beam tracing (LHBEAM) are asymptotic approximations to the wave equation, $k \sim \nabla$ required. => can be problems with boundaries

- Full Wave (TORLH): solves Maxwell's equations directly, yields the electric field.

$$\nabla \times \nabla \times \mathbf{E} = \frac{\omega^2}{c^2} \left\{ \mathbf{E} + \frac{4\pi i}{\omega} \left( \mathbf{J}^P + \mathbf{J}^A \right) \right\} \quad \leftarrow \quad \mathbf{E}(\mathbf{x}) = \sum_m \mathbf{E}_m(\psi) \exp \left( i m \theta + i n \phi \right)$$

\[
\begin{align*}
\frac{dx}{dt} &= -\frac{\partial D}{\partial k} \\
\frac{dk}{dt} &= \frac{\partial D}{\partial \omega} \\
\frac{dP}{dt} &= -2\gamma P
\end{align*}
\]
TORIC Full Wave Code

- TORIC [Brambilla PPCF 1999] was developed with an FLR model for the plasma current response, $J^P$, for ion cyclotron waves and recently extended with an asymptotic form for lower hybrid waves.

- The antenna is modeled as a current sheet, $J^A$ for ICRF and as the mouth of a wave guide with imposed $E_\parallel$ for LH.

- It solves Maxwell's equations for a fixed frequency (Helmholtz problem) assuming toroidal symmetry in a mixed spectral-finite element basis. This is the physical optics solution.

\[
\nabla \times \nabla \times E = \frac{\omega^2}{c^2} \left\{ E + \frac{4\pi i}{\omega} (J^P + J^A) \right\} \quad \leftrightarrow \quad E(x) = \sum_{m} E_m(\psi) \exp (im\theta + in\phi)
\]
A brief history of TORIC

- **FISIC (100x63)** – Brambilla NF 1986, F77, introduced basic algorithm, reduced order for $E_{||}$
- **TORIC (240x127)** – Brambilla NF 1996, F77+F90, completely restructured, full solution
- **TORIC out-of-core (480x255)** – extends problem size
- **TORIC parallel (980x1023)** – F90+MPI+ScaLaPACK, extend problem size, reduces run time
- **TORIC-LH** – Wright CiCP 2004
- + **Fokker-Planck** – Wright PoP 2009
- + **3D processor mesh** (2000x2047)
Lower Hybrid Wave Equation

\[ \nabla \times \nabla \times \mathbf{E} = S \mathbf{E}_\perp + iD (\mathbf{b} \times \mathbf{E}_\perp) + PE_\parallel \mathbf{b} + \nabla_\perp (\sigma \nabla_\perp \cdot \mathbf{E}) \]

- The pressure driven term \( \sigma \) is responsible for the ion plasma LH branch. In regimes of experimental interest it is nearly vanishing.
- We drop this term and solve only for the fast and slow LH waves.
- The dielectric assumes zero FLR electrons and unmagnetized ions (non-Max effects in \( \text{Im} \ P \) retained).
- Residue gives \( \text{Im} \ P \), principle value gives \( \text{Re} \ P \sim Z_0 \).

\[ P \equiv \chi_{zz} = \frac{\omega_{pe}^2}{\omega^2} \int 2\pi u_\perp du_\perp du_\parallel J_0^2 \left( \frac{k_\perp u_\perp}{\Omega e_0} \right) \frac{\partial f/\partial u_\parallel}{1 - k_\parallel u_\parallel /\omega} \]
Scaling/Convergence study

- Progression from 511->1023->2047
- Power deposition broadens at each step
- 18hrs on 2048 cpus on Franklin for $N_m = 2047$
Scaling/Convergence study

- Fully converged on Maxwellian distribution at 2047 poloidal modes.
Single pass is well converged

- Single pass damping on a Maxwellian plasma is converged
- Little interference evident

Power is very localized in single pass.

<- Spectrum is converged at 1023 modes

$n_\parallel = -2.55$, $n_0 = 7 \times 10^{19} \text{ m}^{-3}$
$T_0 = 10 \text{ keV}$, $B_0 = 4 \text{T}$
Cross-code verification/comparison

● Alcator C.
  ~10^6 mesh pts in each simulation. n_\parallel = -2.5, B_0 = 8T,
  n_e0 = 5 \times 10^{19} \text{ m}^{-3}, T_{e0} = 5 \text{ keV}.

Shiraiwa
TI3.03
Meneghini
PP8.03

COMSOL-LH

TORIC-LH

AORSA
Jaeger

Each approach as advantages and difficulties:

• CPU-hrs: COMSOL-LH 13, TORIC-LH 80, AORSA 32k (ray tracing ~ minutes)
  but wall clock times for all three are comparable given number of processors used.

• COMSOL: 2D elements can model wall and separatrix region; sparse matrix scales well [PoP Sep 2009]

• The plasma dielectric:
  - COMSOL-LH: requires real space dielectric formulation, doesn't have algebraic k_\parallel
  - TORIC-LH: FLR truncation efficient for LH
  - AORSA: All Orders treatment most general, can handle fast ion interactions
First ever 3D LH full wave simulation

\[ E_z(r, \theta, \phi) = \sum_{n_{tor}} \frac{P_{LH}}{P_{abs}(n_{tor})} \frac{(P_{ant}(n_{tor}) \times P_{abs}(n_{tor}))}{\sum_{n_{tor}}(P_{ant}(n_{tor}) \times P_{abs}(n_{tor}))} e^{i(\phi n_{tor} + \angle_{ant}(n_{tor}))} E_{z,n_{tor}}(r, \theta) \]

- Sum 100 \( n_\phi \):

Alcator C
f=4.6 GHz, \( n_{||}=2.1 \)
H plasma, 8T, 600kA,
5x10\(^{13}\) cm\(^{-3}\), 3.8 keV
(240,127,100)x6=10\(^7\) DoF
Model plateau \([2.5,8]_v\)\(_{th}\)
J-P. Lee with VisIt
Weak damping requires self-consistent $f(v)$ for damping

- The poloidal power spectrum shows lack of convergence at outer flux surfaces,
- and magnification of unit amplitude applied field.
Simulation model is a coupled full wave and Fokker-Planck system

CQL3D (Harvey 1992 IAEA)

Bounce averaged Fokker-Planck code that solves for new distribution function from RF quasilinear flux, provides $f(v)$

TORIC-LH (Wright 2004 CPC, 2009 PoP)

Dielectric for LH uses zero FLR electrons and unmagnetized ions. Calculates the rf fields and the quasilinear flux.

• Simulations use EFIT reconstructed magnetic equilibria.

• Electron distribution functions from iteration with a Fokker-Planck code are used for dielectric – a non-linear solution.
Hard X-ray gives indirect measure of LH CD

- HXR camera measures bremsstrahlung emissions from electrons accelerated by LHCD.
- Better for simulation comparison than current profile – harder to measure, longer time response.
Non-relativistic iteration is not sufficient

- Broad plateau formation observed.
- Pitch angle scattering creates high energy tails $\sim> 500$ keV
- Synthetic HXR from CQL3D/TORICLH is narrower and weaker than experimental measurement. Total current is 300 kA vs 700 kA in experiment. What effects are missing?
Relativistic effects in dielectric

- Relativistic consistency:
  - Dql from TORIC-LH is fully relativistic,
  - CQL3D evolves the relativistic distribution function,
  - Formulation of parallel dielectric response for general relativistic non-Maxwellian retains the principle value in the form of the Maxwellian Z-fn, Im part has resonance along a hyperbolic line in u-space.

\[
\chi_{zz} = \frac{\omega_{pe}^2}{\omega^2} n_\parallel \int 2\pi u_\perp du_\perp u_\parallel J_0^2 \left( k_\perp u_\perp / \Omega_{e0} \right) \frac{\partial f}{\partial u_\parallel}
\]

\[
u_\perp^2 = (n_\parallel^2 - 1)u_\parallel^2 - 1
\]
Relativistic effects on DqI

- Inclusion of relativistic effects introduces the hyperbolic resonance condition:

\[ \omega = k_{||} v_{||} \Rightarrow \omega = k_{||} \frac{u_{||}}{\gamma} \Rightarrow \]

\[ (n_{||}^2 - 1) \frac{u_{||}}{c^2} - u_{\perp}^2/c^2 = 1 \]

\[
B = \sum_{m,m'} \frac{u_{||}^4}{\gamma^2 \nu_\phi(m) \nu_\phi(m')} J_0^2 \left( k_{\perp} \frac{u_{\perp}}{\Omega_e 0} \right) E_{||}(m) E_{||}^*(m') e^{i \theta(m-m')} \delta(\omega - k_{||}^{(m)} v_{||})
\]

\[
\langle B \rangle \equiv \frac{1}{\tau_b} \oint \frac{B}{v_{||}} d\ell
\]
Experimental validation

- Using the Alcator C-Mod shot #1060728011.1100. We calculate and compare the HXR spectrum with a synthetic diagnostic.
- Using the fields generated from a Maxwellian dielectric we get agreement in strength but not detailed shape profile. Code now matches experimental current magnitude.
2nd Iteration with $X_{zz}$ relativistic

- Hollowness remains in HXR. Fields are more damped.
- Narrowness not understood.
- Adding radial pinch velocity may help fill in center $\sim 1\text{m/s}$
  - Possible non-linear interaction between toroidal modes
Current Density

- Total current is 680 kA.
Summary

- We have developed a tool that for the first time can produce self-consistent simulations of lower hybrid current drive in toroidal geometry using the full wave approach and 3D Fokker-Planck.
- Spectral broadening effects due to diffraction and poloidal coupling are included in the model.
- The full wave calculation yields the electric fields directly.
- Wave fronts are calculated properly near caustics and cutoffs. This may be important in the multi-pass damping regime.
- Improvements in algorithms and computation platforms have both been important in making 3D full wave LH possible.
- Cross code verification and experimental validation have been essential in development.
- Computations are cpu-intensive – new solver (Indireshkumar GP8.052) is 4 to 10 x faster.
- Next steps: non-linear coupling of toroidal modes, analysis of strong absorption experimental cases.
Comparison with ray tracing, weak damping

- Maxwellian damping in both cases.
- Good agreement in power deposition locations.
- In full wave, we see the field amplitudes are magnified by “cavity effect” [Q~200] from unit applied field at guide.
  - Indicates need for non-Max damping and possible edge damping.
Participants in the RF-SciDAC


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CPU requirements are substantial

• Typical resolutions of 1000x1023.
• The stiffness matrix, $A$ is block tridiagonal with the blocks being $(6 \times N_m)^2$:

$$
A = \begin{pmatrix}
D & U & 0 & 0 & 0 \\
L & D & U & 0 & 0 \\
0 & L & D & U & 0 \\
0 & 0 & L & D & U \\
0 & 0 & 0 & L & D
\end{pmatrix}
$$

• Current approach distributes 3 blocks over all processors and uses the serial Thomas algorithm.
• Existing parallel tridiagonal solvers do not distribute the blocks.
New solver has 3D decomposition

Current Solver

\[
L_i \cdot \tilde{x}_{i-1} + D_i \cdot \tilde{x}_i + R_i \cdot \tilde{x}_{i+1} = \tilde{y}_i
\]

\[
D_{i+1} = D_{i+1} - L_{i+1} \times D_i^{-1} \times R_i
\]

\[
(6N_m) \times (6N_m)
\]

New Solver

\[
A x = \begin{bmatrix}
    a_1 & b_1 \\
    c_2 & a_2 & b_2 \\
    & c_3 & a_3 & b_3 \\
    & & \ddots & \ddots & \ddots \\
    & & & b_{n-1} \\
    & & & c_n & a_n
\end{bmatrix}
\begin{bmatrix}
    x_1 \\
    x_2 \\
    x_3 \\
    \vdots \\
    x_n
\end{bmatrix} = \begin{bmatrix}
    r_1 \\
    r_2 \\
    r_3 \\
    \vdots \\
    r_n
\end{bmatrix} \equiv r.
\]

• Serial (Radial direction [i=1..N_ψ]: Thomas Algorithm)
  +
  
2-D Parallel (Poloidal m modes : Scalapack matrix calculation
  \((6N_m) \times (6N_m)\))

1-D Parallel (Radial direction: combination of Divide-and-Conquer
  and Odd-even cyclic reduction Algorithms) \(\rightarrow\) # P1 groups


+ 

2-D Parallel (Poloidal m modes : Scalapack matrix calculation )
\(\rightarrow\) #P2*P3 processors

\(=\) 3-D grid(P_tot=P1*P2*P3)
Strong scaling of old and new solvers to 16k processors

- Time shown only for solver step. Fill in step has slope $\sim -1$.
  2-4 x faster than Thomas solver.
- Old solver (in red) saturated because of complete domain decomposition, not communication.