Investigation of ion toroidal rotation induced by Lower Hybrid waves in Alcator C-Mod* using integrated numerical codes

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Abstract

Ion toroidal rotation in the counter current direction has been measured in C-Mod during lower hybrid (LH) RF power injection. Toroidal momentum input from the LH waves determines the initial increase of the counter current ion toroidal rotation. Due to the fast build up time of the plateau (<1msec), the electron distribution function is assumed to be in steady state. We calculate the toroidal momentum input of LH wave to electrons by iterating a full wave code (TORIC-LH) with a Fokker Plank code (CQL3D) to obtain a self consistent steady state electron distribution function. On the longer time scale, comparable to the transport time (~100msec), ion rotation is changing due to the constant momentum transfer from electrons to ions and the radial flux of ion toroidal momentum by Reynolds stress and collisional viscosity. We suggest a way to evaluate the viscosity terms for the low flow level rotation by a modified electrostatic gyrokinetic code.

Background: LH Wave induces Current and Ion Rotation for Confinement and MHD Stabilization in a Tokamak

- Off-axis Current Drive (Magnetic Shear)
- Supplement bootstrap current
- ITB optimal Control
- Lower Hybrid Wave
- Ion Toroidal Rotation

MHD [tearing, Sawtooth] stabilization

(A.Ince-Cushman et. al. [1])
C-Mod Experimental Toroidal Rotation Results

- Toroidal rotation radial profile measured by High Resolution X-ray spectroscopy (HiReX)
  - Top: LH wave on (t=0.7 sec)
  - Bottom: LH wave off (t=1.2 sec)

- To explain this rotation profiles, it may need
  - Off-axis LH wave momentum source
  - Viscosity for the steady flow
  - Momentum diffusive term
  - Inward convective terms (Sometimes Outward)
  - The effects of temperature and density on the temporary flow

- Lower Single Null (Divertor) magnetic configuration

- Low emissivity of X-ray beyond R~80 cm reduces signal to noise ratio

(R. Parker et. al. [3])
Cmod 1080320017 shot
Temporal ion rotation profile in the core plasma may be related with the density and the temperature.

- $n_e$, $T_e$ are measured by Thomson scattering diagnostics
- $T_i$ is measured by X-ray spectroscopy
- Main ion density (Deuterium) should be lower than electron density due to impurities ($Z_{\text{eff}} \sim 3$)
• Lower hybrid waves ($n_\parallel = -1.9$) induces an ion counter-current toroidal rotation (Non-thermal electrons push ions)
• The summation of electron and ion equations, and the quasi-neutrality make two terms:

\[
\frac{\partial f_e}{\partial t} + \vec{v} \cdot \nabla f_e + \left( \frac{e}{m_e} \nabla \Phi + \Omega_e \vec{\omega} \times \vec{b} \right) \cdot \nabla_v f_e = C_{ee}\{f_e\} + C_{ei}\{f_e,f_i\} + \nabla_v \cdot \vec{D}_{qi} \cdot \nabla_v f_e
\]
\[
\frac{\partial f_i}{\partial t} + \vec{v} \cdot \nabla f_i + \left( -\frac{ze}{M_i} \nabla \Phi + \Omega_i \vec{\omega} \times \vec{b} \right) \cdot \nabla_v f_i = C_{ii}\{f_i\} + C_{ie}\{f_i,f_e\},
\]

Transport Analysis

\[
\frac{\partial}{\partial t} \langle R_i M_i \vec{V}_i \cdot \vec{\zeta} \rangle_\psi = -\frac{1}{v'} \frac{\partial}{\partial \psi} \left( V' \Pi \right) + \langle \vec{f} \cdot \nabla \psi \rangle_\psi - \langle \int d^3v (m_e R\vec{z} \cdot \vec{D}_{qi} \cdot \nabla_v f_e) \rangle_\psi
\]

Wave Analysis

Where \( \langle \ldots \rangle_\psi = \frac{1}{v'} \int d\Theta d\zeta (\vec{B} \cdot \nabla \Theta)^{-1} (\ldots) \), \( V' = \int d\Theta d\zeta (\vec{B} \cdot \nabla \Theta)^{-1} \), and

\[
\Pi = M_i \langle \int d^3v f_i R(v \cdot \vec{\zeta}) (v \cdot \nabla \psi) \rangle_\psi
\]
Three Time Scales in this Analysis

(1) The plateau build up time scale of the electron distribution fn. (< 1msec)
   - Involved with electron Landau damping by LH wave and electron collisions

(2) The transport time scale of the ion rotation (≈100msec)
   - Involved with ion response by the off-diagonal ion stress tensor with ion collisions
   - Neoclassical results + Beating (or transport time averaging) of the gyrokinetic results

(3) The turbulence saturation time scale (~msec)
   - Involved with gyrokinetic response of electron and ion by a perturbed electrostatic potential
     and a short wavelength (<mm) drift wave, and a saturation of its instability.

Assumptions of this analysis

- The electron distribution function \(f_e\) in (1) is independent (it is not affected by the results in the different time scale (2) and (3))
  - Use only wave simulation and electron Fokker-Flank code to obtain \(f_e\).
- The ion dist. fn. in (2) is affected by (1) only through ion-electron collisions
Full Wave Solution with Non-Maxwellian Electron Distribution Function.

- **Maxwell Equations of TORIC-LH**
  - Assume one fixed toroidal mode \( n \), and small electron Larmor radius.

\[
\nabla \times (\nabla \times E) = \frac{\omega^2}{c^2} \left( E + \frac{4\pi i}{\omega} J^p \right)
\]

\[
E(r) = \sum_m E^m(\psi)e^{im\theta+in\phi}
\]

\[
k_\| = \frac{m}{N_r} \sin \Theta + \frac{n}{R} \cos \Theta
\]

\[
J^m(\psi) = \sum_m \sigma(k_\|, \psi) \cdot E^m(\psi).
\]

- **Quasilinear diffusion coefficient for non-maxwellian dis.function.**

  - Define the relativistic Fokker-Plank Eqn,

\[
\frac{dF}{dt} = C(F) + Q(F)
\]

in terms of the outer-midplane distribution function, \( F \), and,

\[
Q(F) \equiv \frac{\partial}{\partial u} (D \frac{\partial F}{\partial u}), \quad u = p/m = \gamma mv, \quad \gamma = \frac{1 + u^2}{2} = (1 - (v/c)^2)^{-1/2}
\]
Momentum Input Calculation in TORIC-LH

• **Bounce averaging power absorption by two methods (Eqn. (1) and (2))**
  
  - Power absorption can be defined with bounce time, \( \tau_b(\varepsilon, \mu) = \int \frac{dt}{\nu_{||}(E, \mu, t)} \)

  
  \[
P_{abs}(\psi) \equiv \int d^3u \gamma mc^2 Q(F) = - \int d^3u (mc^2u/\gamma) \cdot \langle D_{ql} \rangle \cdot \frac{\partial F}{\partial u} \tag{1}
\]

  \[
P_{abs}(\psi) \equiv \frac{1}{2} \sum_m \sum_{m'} \text{Re} \left\{ J^{m'*} \cdot E^m \right\} \sim \frac{1}{\tau_b} \text{Re} \left\{ \sum_m \sum_{m'} \int \frac{d\ell}{|v_{||}|} e^{(m'-m)\theta} E_{||}^{m'*} \text{Im}\{\chi_{||}\} E_{||}^m \right\} \tag{2}
\]

  where \( \langle D_{ql} \rangle \propto \hat{e}_{||} \hat{e}_{||}, \ \chi_{||} = \sigma_{||} \frac{4\pi i}{\omega} \), and

  \[
  \text{Im}\{\chi_{||}\} = -2\pi^2 m_0 (\frac{\omega_p e}{k_{||}})^2 \int_0^\infty dp_{\perp} f_{\perp} \left( \frac{k_{||} p_{\perp}}{m_0 \Omega_0} \right) \frac{1}{\partial v_{||} / \partial p_{||}} \left| v_{||} = \omega / k_{||} \right|
\]

• **Similarly for the toroidal angular momentum input calculation**

  \[
  \frac{dL_\phi(\psi)}{dt} \equiv \int d^3u (mu_\phi c R) Q(F) = \int du^3 (-mcR \hat{e}_{||} \cdot \hat{e}_\phi) \langle D_{ql} \rangle \frac{\partial F}{\partial u_{||}} \tag{3}
\]

  \[
  \equiv \frac{1}{\tau_b} \text{Re} \left\{ \sum_m \sum_{m'} \int \frac{d\ell}{|v_{||}|} e^{(m'-m)\theta} E_{||}^{m'*} \text{Im}\{\eta\} E_{||}^m \right\}
\]

  \[
  \text{Im}\{\eta\} = \frac{k_{||}}{\omega} R(\hat{e}_{||} \cdot \hat{e}_\phi) \text{Im}\{\chi_{||}\} = \frac{n_{||}}{c} R \cos \Theta \text{Im}\{\chi_{||}\} \tag{4}
\]
Iterations between TORIC-LH and CQL3D

- **Self-consistent electron velocity distribution**

  CQL3D (Harvey 1992 IAEA)
  Bounce averaged Fokker-Planck code that solves for $F(v)$

  TORIC-LH (Wright 2004 CPC, 2009)
  Calculates RF fields and the bounce avg. quasilinear flux $(D_{ql}dF/dv)$ by $f(v)$

Velocity Distribution Fn.

![Velocity Distribution Fn.](image)

Quasilinear DiffusionCoeff.

![Quasilinear DiffusionCoeff.](image)
Iteration Convergence in terms of Power absorption profiles

- Iterations until Step 8 profiles almost agree by Python scripts

The dist. Func. at $r/a=0.7$ in CQL3D of the step8

The poloidal spectrum in TORIC-LH of the step8
Electric field of Lower Hybrid Wave in TORIC-LH

2D E field with $n_\parallel = -1.9$

Radial power absorption profile by 3D reconstruction
Computation of Toroidal Angular Momentum Source of Lower Hybrid Wave

- **A radial profile of toroidal angular momentum increase rate (1-D)**
  - Experimental result from the instant change of toroidal rotation and electron density when LH wave turns on (yellow line) (Assumed $n_D = n_e$)
  - Simulation result from the step 8 of TORIC-LH to evaluate $\frac{dL_\phi}{dt}$ in (3) of the slide 9 with one toroidal mode, $n_\parallel = -1.9$ (blue line)

![Graph showing toroidal angular momentum increase rate](image)

- **Total total angular momentum input-Torque (0-D)**
  - Experimental result(yellow line integration) = 0.00929 N·m
  - Simulation result(blue line integration) = 0.00419 N·m
  - Wave momentum input in [3] by $\dot{P}_\phi = \int d\vec{A} \cdot \frac{1}{\omega} \vec{s} \hat{k} \cdot \hat{\phi} = \frac{n_\phi}{c} \int dA \langle W \rangle \vec{v}_g \cdot \hat{\phi} = \frac{n_\phi}{c} Power = 6.7 \times 10^{-3} N\cdotm$
Toroidal Angular Momentum Transport

- The toroidal angular momentum transport eqn.  \[ \frac{\partial L_{Ang}}{\partial t} = \frac{1}{r} \frac{\partial}{\partial r} (r \Pi_{Ang}) \]

- Low flow gyrokinetics to evaluate the radial flux, \( \Pi = M_i \left( \int d^3v f_i R(v \cdot \hat{\zeta})(v \cdot \nabla \psi) \right)_\psi \)
  -(F.I.Parra and P.J Catto in [4])

- Order of the radial flux of toroidal angular momentum
  - \( \delta = \) the ratio of ion gyroradius to the characteristic length(~minor radius)<<1
  - GyroBohm Diffusion \( D_{gB} = \frac{\rho kT}{eB} = \delta \rho v_{th} \)
  - Low toroidal rotation(w/o NB) \( V_i \sim \delta v_{th} \)

  \[ \Pi = M \left( \int d^3v f_i R(v \cdot \hat{\zeta})(v \cdot \nabla \psi) \right)_\psi \sim |\nabla \psi| D_{gB} \times \nabla (RMn_i V_i) \sim \delta^3 p_i R |\nabla \psi| \]

- The evaluation requires \( (B/B_p) \delta^2 f_{Mi} \) -order ion distribution function which is available by modifying the current gyrokinetic code and neoclassical code
Numerical Evaluation of Radial Flux of Toroidal Angular Momentum (planning..)

- 1\textsuperscript{st} $\delta$ order Neoclassical Contribution ($F_{i1}^{nc}$, $\phi_{1}^{nc}$) from "NEO" by solving a neoclassical equation
- 1\textsuperscript{st} $\delta$ and 2\textsuperscript{nd} $\delta$ order Turbulence Contribution ($F_{i2}^{tb}$, $\phi_{1}^{tb}$, $\phi_{2}^{tb}$) from "modified GS2 suggested in [4]"
  - In the gyrokinetic equation, replace the maxwellian $F_{0}$ with $F_{i} = f_{Mi} + F_{i1}^{nc}$
- 2\textsuperscript{nd} $\delta$ order Neoclassical Contribution ($F_{i2}^{nc}$) from "modified NEO"
- Transient pressure contribution ($\frac{\partial p_{i}}{\partial t}$) from experimental results

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{diagram}
\caption{A diagram to find the radial E field: numbers in the parenthesis means the equation index in [4]}
\end{figure}
Radial Flux of Toroidal Angular Momentum with Lower Hybrid Wave

\[
\frac{\partial}{\partial t} \langle R_n M_i \tilde{V}_i \cdot \hat{\zeta} \rangle_\psi = -\frac{1}{v'} \frac{\partial}{\partial \psi} (V' \Pi) + \langle \tilde{J} \cdot \nabla \psi \rangle_\psi - \langle \int d^3v (m_e R^\zeta \cdot \overline{D_{ql}} \cdot \nabla \nu f_e) \rangle_\psi,
\]

\[
\Pi = \frac{M c}{2Z e} \left[ \frac{\partial}{\partial t} \langle R^2 \hat{\zeta} \cdot \overline{P_i} \cdot \hat{\zeta} \rangle_\psi \right] + \frac{M^2 c}{2Z e} \frac{1}{V'} \frac{1}{\psi} V' \left[ \langle \int d^3v f_i R^2 (v \cdot \hat{\zeta})^2 (v \cdot \nabla \psi) \rangle_\psi \right] + \frac{\partial \phi}{\partial t} \left[ \frac{\partial}{\partial \zeta} \langle R_n M_i \tilde{V}_i \cdot \hat{\zeta} \rangle_\psi \right] - \frac{M^2 c}{2Z e} \left[ \langle \int d^3v (C_{ii}\{f_i\} + C_{ie}\{f_{i0}, f_{e1}\}) R^2 (v \cdot \hat{\zeta})^2 \rangle_\psi \right].
\]
Issues and Future Plan

Wave Part

• Numerical Iteration Convergence
  – More careful control of the iterations between TORIC-LH and CQL3D
  – Increased resolution in real and velocity space of TORIC-LH (new solver required [5])
  – 3-D reconstruction with more toroidal modes (new solver required)

• Transform from bounce averaging to flux surface averaging

• Theoretical Verification
  – Validation of the quasi linear diffusion coefficient in the toroidal geometry in terms of non-linear mode coupling

• Experimental Data
  – Reliability of the X-ray spectroscopy rotation data (Low emissivity in the plasma edge)
  – Verification of current drive by lower hybrid wave using MSE

Transport Part

• Inward momentum pinch?
• Quasi-neutrality until 4th order in $\delta$?
• Order analysis reality (GyroBohm, low flow)
• LH wave effect on ions ($C_{ii} \rightarrow C_{ii} + C_{ie}$)
• Transform of dist. fn. in terms of coordinate, $\langle r, E_0, \mu_0, t \rangle \leftrightarrow \langle R, E, \mu, t \rangle$
• Up-down symmetry assumption
• Numerical analysis accuracy and sensitivity
Summary

- In Alcator C-Mod, ion toroidal rotation in the counter-current direction induced by lower hybrid wave (LH) was measured with lower single null magnetic configuration in 2008.

- Toroidal angular momentum input of LH wave to electrons is evaluated by iterating a full wave code (TORIC-LH) with a Fokker Plank code (CQL3D) for the self-consistent electron distribution function in the plateau build up time scale (<1msec). Also, the simulation result is compared with the experimental increase rate of the angular momentum when LH wave turned on.

- A quantitative framework to evaluate the off-diagonal stress tensor (or the radial flux of toroidal angular momentum) is suggested to explain ion toroidal rotation in the transport time scale (~100msec) based on the low flow gyrokinetics in [4].