Formation and stability of impurity-induced \((m,n)=(1,1)\) snakes in Alcator C-Mod

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Motivation for new studies of snakes

Longed-lived helical snake-like structures were discovered two decades ago and have been observed in every major magnetic fusion facility.

• However, the formation of the snake, the reasons for its improved stability and its impact on the background plasma are still unknown.

• Today, thanks to new diagnostics with adequate spatial resolution (AXUV bolometers and x-ray crystal imaging spectrometer) that supplement SXR tomography, the role of $n_z$, $\nu_e$, $\eta$, $\nu_\phi$ and $P_{rad}$ can be studied for the first time.
Snakes were first seen at JET (A. Weller, PRL ‘87) as a persistent density perturbation at a rational \( q \)-surface following the injection of a fueling pellet. Since then, snakes have been observed in Alcator C, Alcator C-Mod, ASDEX-U, CDX-U, Globus-M, HL-1M, JET, JT60-U, NSTX, TEXTOR, TFTR, Tore-Supra, etc.

A *snake-like* \( 1/1 \) helical pattern is characterized by a small region of localized and enhanced SXR emission that rotates within the field of view of the SXR arrays and is radially concentrated on, or inside the \( q=1 \) surface. SXR reconstruction showed the localized nature of the perturbation.

The density perturbation has an “infinite” particle/impurity confinement time. Snakes persisted for surprisingly long time, surviving several sawtooth crashes.

Its formation and stability mechanisms have never been elucidated.

It is not clear yet what is the role of the impurity density & resistivity in the formation and stability of the snake.
Pellet-induced snake on JET (or JET-like):

a. A deuterium fueling pellet reaches the $q=1$ surface, cooling each flux surface that intersects its trajectory, but the strongest effect occurs on the shortest field lines at $q=1$.

b. **Conventional interpretation:** Pellet causes localized cooling on the $q=1$ surface that increases the local resistivity, causing a drop in current density ($J_\phi \sim E_\phi / \eta$) that then leads to the formation of a [nonlinear] $(m,n)=(1,1)$ magnetic island that “traps” the particles from the pellet.

c. The “snake” density was much higher than its surroundings and persisted for surprisingly long times, surviving even several sawtooth-crashes!

d. Perturbations were typically localized to $\sim 10$-50% of the poloidal circumference of the $q=1$ surface, having a radial width somewhat smaller than the poloidal extent ($l_r < l_\theta$).
(1,1) snakes in C-Mod have been observed using the x-ray tomographic (XTOMO) arrays.

1. XTOMO signals suggest that spontaneous, large impurity injections occur at discrete times near the beginning of ohmic discharges (t=0.2-325 s).

2. Data from XTOMO cameras indicate enhanced SXR brightness from plasma cooling or penetration of impurities (t=0.325 s).

3. Last high-Z injection @ ~ 0.325 s cools the plasma core shortly before snake formation.
Impurity injection causes only small $T_{e0}$ drop

① The snake forms shortly after the peaking of the bolometer radiated power and the drop of the central electron temperature.

② Increase to 3.7 MW/m³ peak radiated power density can be associated with radiation due to a high-Z impurity. First hints of impurity asymmetry are visible from the $P_{\text{rad}}$ profile at $t=0.334$ s.

③ Temperature drop is small: $\sim10\%$ or $\Delta T_{e,0}\sim120$ eV.
An \( m=1 \) perturbation is observed in both the vertical and horizontal XTOMO cameras. The “snake-like” pattern in the SXR brightness is apparently formed by an enhancement of the impurity density in a small helical “flux tube”, on or inside the \( q=1 \) surface that rotates toroidally within the field of view of the SXR arrays.
High-Z impurity density in the snake decreases gradually in time

① SXR show a well localized, asymmetric region of stronger emissivity centered on a radius $r=r_s$, superimposed on a centrally peaked background emissivity.

② The asymmetry resembles the island location in a 1/1 helical kink structure, as shown by the profiles of high- vs. low-field side (inboard vs. outboard) peaks.

③ The center of the helical structure moves radially outwards in time.

④ The impurity concentration in both the background core and the helical structure decreases over time.
High resolution x-ray imaging spectrometer shows that molybdenum is responsible for the snake impurity concentration in Alcator C-Mod.
Alcator C-Mod x-ray crystal imaging spectrometer is designed to view the entire poloidal cross section.

Plasma height ~ 70 cm
Detector height ~ 27.5 cm
De-magnification ~ 2.5-2.6
~ 1cm chordal resolution

C-Mod spectrometer design criteria:

1. From Doppler line-widths and -shifts ⇒ infer \( T_i \) and \( v_\phi \) profiles.
2. Temperature: \( 0.1 \text{ keV} < T_i \& T_e < 6 \text{ keV} \): measures He-like \& H-like argon spectra.
3. Sufficient spectral resolution to measure rotation velocities with an accuracy better than \( \sim 5 \text{km/s} \) (\( \lambda / \Delta \lambda \sim 10^4 \)).
4. Sufficient temporal resolution for impurity transport studies (\( \Delta t \sim 5-20 \text{ ms} \)).
5. Tomographic inversion is possible under a certain set of assumptions.
Almost all the ionized Mo in the core of Alcator C-Mod is Ne-like Mo (Mo32+)

1. Mo is the main high-Z intrinsic impurity at C-Mod since more than 95% of the plasma facing components are made out of pure Mo and/or TZM (99% Mo).

2. The average molybdenum ion charge for core electron temperature of interest is $\langle Z \rangle \sim 32$.

3. Data from XTOMO and AXUV bolometers, with known cooling factors, can be used as a proxy for $n_{Mo}/n_e$:

$$ L_{Mo} \sim 7 \times 10^{-32} \text{ W} \cdot \text{m}^3 $$

Mo32+ (neon-like) line falls into H-like Ar spectrum

Ar - Lyα1, Lyα2 and Mo32+

\[ \frac{v_{th, Mo}}{v_{th, Ar}} \sim \sqrt{m_{Ar}/m_{Mo}} = 0.65 \]
Molybdenum from the metal walls is responsible for the enhanced SXR and $P_{\text{rad}}$. Brightness of Mo$^{32+}$ peaking at the core before “snake” is observed.
Enhanced SXR brightness is well correlated with an increase in Mo$^{32+}$ line intensity. Conclude that the “snake-like” pattern in the brightness data is formed by a small region of localized and enhanced molybdenum density.
Snake formation: central impurity $\rightarrow$ kink asymmetry $\rightarrow$ strongly kink circular core $\rightarrow$ crescent island

① Centrally peaked brightness immediately after the last Mo injection (initially, $q_0>1$).

② Kink-like asymmetry in Mo density distribution develops.

③ Structure resembles the displaced circular core of an internal resistive $(m,n)=(1,1)$ kink, rotating with the plasma toroidal flow velocity.

④ Kink structure grows in radius.

⑤ 1<sup>st</sup> possible “sawtooth crash” defined as a detectable SXR pulse propagating radially outwards.

⑥ Snake-like structure evolves radially narrower but with a wider poloidal extent (like crescent island in 1/1 kink).

Graph:
- Time (s)
- Chord #
- Centrally peaked core
- Snake formation
- 1<sup>st</sup> & 2<sup>nd</sup> sawtooth crashes
- Outboard
- Inboard
- r/a~0 core sightline
Kink-like asymmetry in the Mo impurity density develops soon after the final impurity injection.

1. Initial core emissivity is axisymmetric and peaked at the magnetic axis (t=0.320 s).
2. The core emission increases 250 kW/m³ after the last Mo injection.
3. Peak develops 1/1 kink-like displacement from magnetic axis (δr~1 cm, t=0.334 s).
4. The SXR asymmetry could be as high as 300 kW/m³ in agreement also with bolometric measurements.
Snake formation is accompanied by kink-like off-axis impurity density perturbation ($\delta n_{Mo}$).

1. Small changes in the SXR tomography and AXUV bolometers indicate a small (<1 cm) radial displacement of emissivity peak, within 0.5 ms before the first observable poloidal excursion (see first panel on the top).

2. Perturbed impurity asymmetry before the first sawtooth crash resembles that of the displaced circular core of an internal resistive kink.

3. The crescent island-like structure of the later snake forms after the first sawtooth crash(es), 2 ms or more after the first snake first appears.
Later snake: SXR tomographic reconstruction shows 1/1 crescent shape

① Dimensions: \( r_s \sim 5 \text{ cm}, \) full-width \( \omega_{\text{sat}} \sim 6 \text{ cm} \) and \( \pi < \sigma_0 < 5\pi/4 \) radians.

① Perturbation travels in the electron diamagnetic drift direction (consistent with \( v_\phi \)).

② From the transit time \( \tau \sim 185-200 \ \mu\text{s} \) we calculate \( v_\phi \sim 21-23 \text{ km/s} \), which agrees with the toroidal velocity measured from the x-ray line Doppler shifts.
Snakes are surprisingly stable: they survive multiple sawtooth crashes

\[ \tau_{\text{snake}} \sim \tau_{\text{rot}} \sim 185-200 \, \mu s \text{ while } \tau_{\text{sawtooth}} \sim 3.66 \, \text{ms} \]

⇒ approximately 20 snake periods between sawtooth crashes (see white arrows).

1. \( \tau_{\text{snake}} \sim \tau_{\text{rot}} \sim 185-200 \, \mu s \) while \( \tau_{\text{sawtooth}} \sim 3.66 \, \text{ms} \)

2. Crashes may cause a transient reduction of \( r_{q=1} (\delta r_1 \leq 1 \, \text{cm}) \).

3. Each crash contributes to flushing high-Z core impurities from both the background core and the snake.
Sawtooth crash flushes impurities from snake

① SXR emissivity $\propto n_{Mo}$ decreased from 1600 to 750 kW/m$^3$ in 50 ms.

② Circular core displaced in time to the original magnetic axis.

③ The poloidal area covered by the snake also increases in time due to the radial penetration of the plasma current and the expansion of $r_{q=1}$. 
Circular “dark” core moves radially outwards during sawtooth crash

① The sawtooth crash is apparently due to the circular core moving radially outwards while the impurity island responds to these changes.

① The poloidal area covered by the snake transitions from a bean- to a D-shaped kink during the crash, increasing both the $n_{M0}$ and the emissivity inside the helical island.

① When the plasma recovers, the circular core moves back radially inward, reinstating the the bean-like impurity 1/1 helical island.

① The circular core, with fewer impurities, heats faster until the pressure drives another incomplete sawtooth crash.
The snake perturbation rotates toroidally at the same speed as the background plasma.

① The plasma toroidal rotation speed is roughly constant over the snake lifetime.

② Additional evidence for a snake magnetic structure.
Central electron temperature profile is not affected by the presence of the mode
New bolometric reconstructions of $\delta n_Z$ from $\delta P_{\text{rad}}$ show strong impurity concentration

Do $\delta P_{\text{rad}}$, $\delta n_Z$, $\delta Z_{\text{eff}}$ have an effect on the formation and stability of the mode?

Previous snakes studies relied only on bolometric and SXR estimates from which it was difficult to infer local values of $\delta n_Z$, $\delta Z_{\text{eff}}$, and $\delta \eta$. 

\[ \delta n_{Mo} \sim \frac{\delta P_{\text{rad}}}{n_{e,0} L_{Mo}} \]

Estimate:

- $P_{\text{rad}}^{\text{snake}} \sim 3.75 \text{ MW/m}^3$
- $P_{\text{rad}}^{\text{back}} \sim 1.45 \text{ MW/m}^3$

$t_0 = 340.902 \text{ ms}, \theta \sim 0$

$t_1 = 340.993 \text{ ms}, \theta \sim \pi$

$T_{e0} \sim 1.41 \text{ keV}$
Bolometric 1D Abel-reconstructions of $\delta P_{rad}$ inside the snake is good match to XTOMO

$R_0 \sim 67-68 \text{ cm}$

$\omega_{sat} \sim 1-1.5 \text{ cm}$

$r_s \sim 5 \text{ cm}$

$P_{rad, BOLO}$

$(\theta \sim 0, t = 0.340 \text{ s})$
Snake formation: large $\delta n_Z$ and $\delta Z_{\text{eff}}$

1. Bolometer estimates:

$$P_{\text{rad}, Mo} = n_e^2 \left( \frac{n_{M o}}{n_e} \right) L_{M o}$$

2. $L_{M o} \sim 7 \times 10^{-32} \text{ W} \cdot \text{m}^3$ [K. Fournier, NF, (1997)] for a $T_e \sim 2 \text{ keV}$ and $n_{e,0} \sim 1.5 \times 10^{20} \text{ m}^{-3}$

3. An upper-bound of radiated power at the core before the snake formation is $3.1 \text{ MW/m}^3 \Rightarrow n_{M o} \sim 2.95 \times 10^{17} \text{ m}^{-3}$

$$\alpha_{M o} \lesssim 2.8 \quad Z_{\text{eff}, 0} \sim 4.2$$

4. The radiated power on the center of the kink-like snake right after it is formed is greater than that of the original background by $\sim 0.5 \text{ MW/m}^3$
\( \delta n_Z, \delta n_e, \text{ and } \delta p \) evolve during the later snake

- \( \delta n_e/n_{e,0} \) and \( \delta \eta/\eta_0 \) can be as high as 4.7% and 55%, respectively.
- Sawteeth remove impurities as well as shift \( r_{q=1} \) radially outwards.
- \( \delta p \) and \( \delta n_Z \) inside the island decrease at a different ratio since \( T_{e,0} \) is increasing due to Ohmic heating rate.
What makes the snake so persistent?

Assumption:
The persistence of the snake is due to the fact that it embodies a quasi-steady-state magnetic structure. Its configuration and association with the $q=1$ surface suggests it is a resistive 1/1 kink with a magnetic island.

Extended Modified Rutherford Equation (MRE):

$$\frac{\tau_R}{r_s} \frac{d\omega}{dt} = r_s \Delta' - C_1 \left( \frac{\omega^*}{\omega^{*2} + \omega_\chi^{*2}} \right) + C_2 \left( \frac{\omega^*}{\omega^{*2} + \omega_\chi^{*2}} - \frac{\omega_{\Pi}^{*2}}{\omega^{*3}} \right) + C_3 \omega^* + \frac{C_4}{\omega^*}$$

- Previous discussions of snake formation invoked a MRE for the non-linear evolution of $\Delta'<0$ [Wesson (PPFC), Pecquet (NF) and Liu (IAEA, PPCF)].
- Study the effect of impurities on NTMs (m=2): Delgado-Aparicio (IAEA, NF).
- MRE predicts a dominant role for $\delta Z_{eff}$ and $\delta P_{rad}$ in destabilizing a resistive magnetic island.
- The usual Rutherford $\Delta'$ analysis is not valid for (1,1) modes!
3D equilibria can contain 1/1 surfaces

A. Thyagaraja [PoF, (1991)] showed analytically the existence of an equilibrium MHD 1/1 island for \( q_0 < 1 \) (incompressible MHD approximate solution). With a flat pressure profile, persistence of the 1/1 island requires non uniform resistivity within the island:

\[
\frac{\Delta \eta}{\eta_0} > 0
\]

B. Park et al. [PoF, (1987)], showed numerically the existence of a saturated 1/1 island in compressible MHD if the pressure inside the magnetic island is higher than in the plasma core.

C. Stabilizing term due to reversed axisymmetric \( dp/dr \) appears in cylindrical and toroidal 1/1 linear theory:

Shafranov:

\[
\gamma_{m=1} = \int_0^{r_s} \left( -2 \varepsilon^2 r^2 \frac{\partial P}{\partial r} + \varepsilon^2 r B_\theta^2 (3q + 1)(1 - q) \right) dr
\]

Bussac:

\[
\delta W^{(1,1)} \approx \frac{3 \varepsilon^4}{R_0^4} (1 - q_0) \left( \beta^2_{\theta,c} - \hat{\beta}^2_\theta \right)
\]

\[
\hat{\beta}_\theta = \frac{-2 \mu_0}{B_\theta^2 (r_1) r_1^2} \int_0^{r_1} r^2 \frac{dp}{dr} dr
\]
Experimental data suggest a resistive kink island consistent with both Park and Thyagaraja saturated (1,1) state

- The core unperturbed pressure is, $P_0 = [2n_{e,0} - (Z-1)n_{Z,0}]k_B T$ while the pressure at the center of the snake exceeds that of the core by $\delta P = P_S - P_0 = (Z+1) \delta n_Z k_B T > 0$. The measured enhanced pressure could be as high as 5%.

- The $Z_{\text{eff}}$ and resistivity ($\eta$) at the center of the snake exceeds that of the core by $\delta \eta/\eta_0 = \delta Z_{\text{eff}}/Z_{\text{eff}0} > 0$. The measured enhanced resistivity could be as high as 56%.

- An alternative configuration with a helical core pressure but without reconnection, is that of the ideal internal kink mode structure which resembles that of a circular displacement (different that the crescent island-shape).
Axisymmetric equilibria can have helical pressure distributions

Ideal MHD equilibria with reversed $q$-profiles [Cooper, PRL, (2010)] can have non-axisymmetric pressure profiles without islands, with the peak pressure in the core.

① The helical equilibrium states resemble saturated internal ideal kink mode structures.

② These profiles resemble the early C-Mod snake formation.
Summary

① Impurity-induced $(m,n)=(1,1)$ snakes are observed in Alcator C-Mod ohmic discharges, when molybdenum flakes released from the tiles covering the vacuum vessel reach close to the plasma core.

① A novel suite of spectroscopic diagnostics allow the determination of the $P_{\text{rad}}$, $n_Z$ and $Z_{\text{eff}}$ profiles in the island with unprecedented temporal and spatial resolution. The asymmetric plasma pressure, impurity density and resistivity ($\delta p$, $\delta n_Z$ and $\delta \eta$) has been measured for the first time.

① The quasi-steady-state snake structure changes between its formation (displaced circular core), and its long-term crescent-like shape.

② The condition for a saturated island can not be inferred from an extended Modified Rutherford formalism, but from considering 3D ideal or resistive quasi-steady-state kinked equilibria (with a 3D helical core and axisymmetric boundary conditions).

③ Important questions remain (e.g. enhanced particle confinement).

④ New diagnostics should help (e.g. 3\textsuperscript{rd} XTOMO array, MSE, polarimeter and fast x-ray imaging crystal spectrometer).