Exponential Spectra in Alcator C-Mod Edge Turbulence

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Exponential Spectra and Lorentzian Pulses Are Observed in Edge Plasma of Alcator C-Mod

- Dedicated search for evidence for exponential spectra and Lorentzian pulses in edge turbulence in a tokamak has been conducted.

- Proposed that exponential power spectra seen in edge turbulence of fusion plasmas is the result of Deterministic Chaos, and is associated with the presence of Lorentzian pulses in the time series data [Maggs PRL 2011].

- Using reflectometer and Gas Puff Imaging (GPI) data in the Alcator C-Mod tokamak, we analyzed density fluctuation spectra in Ohmic and L-mode plasmas.

  - Density fluctuations just inside or at the Last Closed Flux Surface (LCFS) exhibit exponential power spectra.

  - Individual Lorentzian pulses in the reflectometer time series are found with widths that match inverse characteristic frequency of exponential spectra, consistent with Deterministic Chaos model of Maggs.

  - Distribution of pulse widths is narrow, also consistent with Deterministic Chaos model of Maggs.
Background
Edge turbulence in tokamaks plays critical role in determining overall performance

- Deeper understanding of edge turbulence, and fundamental underlying physical processes determining transport of heat and particles, is desired

- Turbulence is “grand challenge” in physics

- “Universality” of turbulence across many physical systems observed

Influential work of Kolmogorov -- power law shape of spectra is expected due to algebraic dependencies –

- Tokamak edge turbulence displays power law spectra, but multiple piecewise fits (over limited frequency ranges) needed to match spectra

- Recently, a unifying picture of exponential spectra has emerged in the literature, based on Deterministic Chaos [Maggs PRL 2012]

- Experimental evidence from basic plasma experiments and stellarators show agreement with model over broad frequency range, with one fit to spectrum
Universality of edge turbulence in magnetically confined plasmas explored using power spectra measurements

- Plasma edge fluctuation measurements carried out in different magnetic confinement devices (tokamaks & stellarators) support view that plasma turbulence/transport displays universality

- New theories suggest that measurements of fluctuations exhibiting power spectra with exponential frequency dependence (Lorentzian spectral fit) over a broad frequency range is signature of deterministic chaos
LAPD Experiments: Measurements Across Pressure Gradient Region Provide Evidence Linking Exponential Power Spectra to the Presence of Lorentzian Pulses

Only Lorentzian pulse gives exponential spectrum over large frequency range, as observed in experiment

TJK Stellarator: experimental evidence linking exponential power spectra and Lorentzian pulses

Lorentzian pulses have widths consistent with exponential spectra 1/frequency scaling
Experiments on Alcator C-Mod
Our study focused on the search for Lorentzian pulses and exponential spectra in edge fluctuation data at C-Mod Tokamak.

Light intensity fluctuations from Gas Puff Imaging, \( \tilde{n}/n \)

S(t) = A(t)cos\( \phi \)(t) fluctuations from O-mode reflectometer, \( \tilde{n} \)

1\( \mu \)s snapshot of GPI emission showing turbulence structures

Power spectra of edge fluctuations from reflectometer

V. Winters, 54th Annual Meeting of the APS Division of Plasma Physics, Oct 29th - Nov 2nd 2012 Providence, RI, USA
Reflectometer and Gas Puff Imaging data show that exponential spectra exist robustly in C-Mod edge plasma: straight line when plotted semi-log.

Gas Puff Imaging (GPI) diagnostic

Avalanche Photodiode (APD) data from 2-D array is analysed

Exponential spectra observed both inside and outside Last Closed Flux Surface (LCFS)

O-mode reflectometer

Exponential spectra observed inside LCFS

Exponential fits well over broad frequency range

\[ R \sim 0.900 \text{ m outside LCFS} \]
\[ R \sim 0.895 \text{ m just at LCFS} \]
\[ R \sim 0.890 \text{ m inside LCF} \]
Over 45 Ohmic and L-mode plasmas (Ion Cyclotron Range of Frequency (ICRF) heating) are studied in detail at C-Mod

<table>
<thead>
<tr>
<th>Shot</th>
<th>T1 (ms)</th>
<th>T2 (ms)</th>
<th>Bt (T)</th>
<th>Te local (eV)</th>
<th>RF (MW)</th>
<th>Type</th>
<th>NL_04 (m$^{-3}$)</th>
<th>Radius (m)</th>
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<tbody>
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<td>1000</td>
<td>1100</td>
<td>5.25</td>
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<td>1.88-2.18</td>
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<td>980</td>
<td>5.39</td>
<td>120</td>
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<td>800</td>
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<tr>
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<td>Ohmic</td>
<td>1.01e20</td>
<td>.876±.0024</td>
</tr>
</tbody>
</table>
Measured spectral shape follows exponential fit better than power law fits, with agreement over broad frequency range

- Density fluctuation spectra from reflectometer $S(t) = A(t) \cos \phi(t)$ measured near $r/a = 0.95$ in L-mode

- Spectra are plotted linear-linear, semi-log, and log-log

- Exponential fit (red dashed) is shown in bottom two plots

- Exponential fit agrees with data over wide frequency range

- Narrow peaks near 200 kHz and above 700 kHz are electronics noise
Lorentzian pulses are identified in reflectometer time series, widths agree with characteristic frequencies of spectra

- Pulses are found by manually going through time series data and fitting pulses
- Lorentzians are fit through the CURVEFIT routine in IDL
- Formula that was fitted:
  \[ L(x) = A \frac{\tau}{(x - x_0)^2 + \tau^2} + B \]
  - \(A, B, \tau, x_0\) were parameters determined using the fitting routine
  - \(\tau = 1/(2f_{\text{char}})\)
Many Lorentzian pulses identified

- Edge reflectometer data, $r/a \sim 0.98-1.0$, L-mode (low power, RF $\sim 1-2$ MW)
- > 75 pulses identified in 6ms interval during shot 1120224019
Equally positive and negative going pulses

- Pulses have an equal probability of being positive or negative
- Time series has approx. Gaussian Probability Distribution Function (PDF)
Pulses fit very well to a Lorentzian Shape, but not as well to a Gaussian shape

- Edge reflectometer data, $r/a \sim 0.98-1.0$, L-mode (low power, RF $\sim 1-2$ MW)
- Fitting Gaussian, only half of fits will converge (when no tails in data)
- Lorentzian needed to fit tails seen in pulses in data set

![Graph showing Lorentzian and Gaussian fits with R2 values]
Connection between exp. spectra and pulse width

- **Lorentzian Equation:**
  \[
  L(x) \propto \frac{1/(2f_{\text{char}})}{(x - x_0)^2 + (1/(2f_{\text{char}}))^2}
  \]

- **Corresponding Fourier Transform:**
  \[
  \tilde{L} \propto \exp \left( -\frac{f}{f_{\text{char}}} \right)
  \]

- In this case \(1/f_{\text{char}} = 0.00419\), closely matched to independently fit full pulse width 0.00424 [1/f_char in equation]
Strongest scaling of characteristic frequency is seen with line averaged density
- Line averaged density measured with two color interferometer (TCI)
- [See Cale Kasten’s poster this session JP8.00084 ]
$1/f_{\text{char}}$ is inversely proportional to the local density as well, but dependence is weaker.

- Weak dependence on local (Located at 95% flux surface) $n_e$, gradient of $n_e$

- Local parameters measured with edge Thomson Scattering system
  - See John Walk’s poster this session JP8.00078
Characteristic frequency of exponential spectra shows no dependence on local $T_e$ or $\text{grad } T_e$

- No dependence on local (located 95% Flux Surface) $T_e$, gradient of $T_e$

- Local parameters measured with edge Thomson Scattering system
  
  - See John Walk’s poster this session JP8.00078
Comparisons with Theory and Experimental Results from TJK
TJK Stellarator: The waiting time, $\Delta t$, between pulses exhibits an exponential probability distribution function

$$PDF(\Delta t) \approx f_{wt}e^{-f_{wt}\Delta t}$$

- There are strong similarities between the stellarator data and the new C-Mod data set
  - Exponential spectral shape
  - Distribution of pulse waiting times
  - Distribution of pulse widths
- Very similar results overall from C-Mod and TJK – argument for universality?

Distribution of Waiting Times exhibit exponential shape, consistent with Stellarator results

- Edge reflectometer, $r/a \sim 0.98$

- > 75 pulses identified in 6ms interval during shot 1120224019

- Hornung empirical formula is consistent with C-Mod data, using $f_{\text{wait}} = 12.918 \text{ kHz}$, which is value obtained from TJK data set

- Independent best fit line to C-Mod gives $f_{\text{wait}} = 12.71 \text{ kHz}$ as value obtained from C-Mod data set

![Graph showing distribution of waiting times]
Distribution of pulse widths exhibits narrow distribution, consistent with Deterministic Chaos model

- >75 pulses identified in 6ms time interval in shot 1120224019

- Pulses exhibit a narrow distribution, with half widths equal to $2/f_{\text{char}}$ found from the power spectrum fit

- $1/f_{\text{char}}$ for this shot = .0048

- Linear GYRO analysis shows electron mode DW frequency of 100-500 kHz in edge ($r/a = 0.9$) consistent with $f_{\text{char}}$

Chaotic system would exhibit narrow pulse width distribution

Distribution of pulse amplitudes appears to be random, but more data is needed to confirm

- Edge reflectometer, $r/a \sim 0.98$

- Smaller amounts of low and high amplitudes can be due to noise and sampling rate

- Need more data collected at even higher sampling rates

- An automated fitting routine (e.g. Pace, Hornung and van Milligen) may also help extract pulses with extreme amplitudes
Summary I: Exponential Spectra and Lorentzian Pulses are Observed Routinely in Edge Plasma of Alcator C-Mod

• Using reflectometer and Gas Puff Imaging (GPI) data in the Alcator C-Mod tokamak, we analyzed density fluctuation spectra in Ohmic and L-mode plasmas.

• Density fluctuations just inside or at the Last Closed Flux Surface (LCFS) exhibit exponential power spectra.

• Individual Lorentzian pulses in the reflectometer time series are found with widths that match inverse characteristic frequency of exponential spectra.

• Comparison with Gaussian fits / Lorentzian fits show that Lorentzian fits are better at capturing tails in pulse shapes.

• Characteristic frequency shows strongest scaling with line averaged density, weaker scaling with local density, no scaling with temperature or magnetic field.
Summary II: Comparisons with models suggest that C-Mod edge turbulence exhibits chaotic dynamics

<table>
<thead>
<tr>
<th>Maggs (deterministic chaos)</th>
<th>Terry (dissipation range cascades)</th>
<th>van Milligen (Self Organized Criticality)</th>
<th>Winters experiment (tokamak)</th>
<th>Hornung experiment (Stellarator)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lorentzian Pulses (LP) in time series</td>
<td>n/a</td>
<td>Lorentz./Gauss. pulses in time series</td>
<td>yes</td>
<td>yes</td>
</tr>
<tr>
<td>Power spectra are exponential</td>
<td>Power spectra are exponential at high-f, power law at low-f</td>
<td>Power spectra are exponential</td>
<td>Exponential spectra over broad range of frequency</td>
<td>Exponential spectra over broad range of frequency</td>
</tr>
<tr>
<td>LP width matches 1/f_{char}</td>
<td>n/a</td>
<td>No relation</td>
<td>LP width matches 1/f_{char}</td>
<td>LP width matches 1/f_{char}</td>
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<tr>
<td>LP widths narrowly distributed</td>
<td>n/a</td>
<td>Pulse widths randomly distributed</td>
<td>Narrow distribution</td>
<td>Narrow distribution</td>
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<tr>
<td>n/a</td>
<td>n/a</td>
<td>Waiting times random</td>
<td>Waiting times exponentially distributed</td>
<td>Waiting times exponentially distributed</td>
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<tr>
<td>n/a</td>
<td>n/a</td>
<td>n/a</td>
<td>Pulse amplitudes randomly distributed</td>
<td>n/a</td>
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</tbody>
</table>
Future work will expand data base to allow for more stringent comparisons with different models

• Automated fitting routine
• More pulse data to improve distribution studies
• More extensive look at GPI data and addition of Langmuir probe data
• ?
I-Mode and H-Mode plasmas exhibit partly-exponential power spectra, differing at lower frequencies

- I-mode look like a Gaussian shape (the QCM and the WCM) riding on top of a background exponential

- This is consistent with I-mode results and spectral fitting done by Dominguez [MIT PHD thesis 2012]
Examples of spectra attempted fits to Terry model

- **Fit to equation:** $E_B(k) = k^{-\beta} \exp[-b(k/k_{\text{char}})^\alpha]$

- **Could account for the peak of spectrum at lower frequencies, however decays too slowly.**