Turbulent driven ion intrinsic rotation due to the diamagnetic flows

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Outline

• How does the momentum transport occur in a tokamak?

• Can diamagnetic flows induce turbulent momentum transport?

• Examples of diamagnetic effects on rotations

1. Reversal of rotation direction due to collisionality [Barnes, PRL (2013)]

2. Rotation change in the lower hybrid wave momentum injection

3. Peaked profile of intrinsic rotation in L-H mode transition
Ion toroidal angular momentum is redistributed due to turbulence

- Toroidal momentum transport equation for the toroidal flow \( V_\varphi = \Omega_\varphi R \) is
  \[
  \frac{\partial}{\partial t} \left\langle n_i m_i R V_\varphi \right\rangle_\psi = -\frac{1}{A} \frac{\partial}{\partial r} (A \Pi) + T_\varphi
  \]

- The momentum redistribution is due to the turbulent radial flux of toroidal angular momentum
  \[
  \Pi \approx \left\langle \left( n_i m_i R V_\varphi^{tb} \right) (v_E^{tb} \cdot \hat{r}) \right\rangle_\psi
  \]

- For intrinsic rotation (rotation without torque), we need to understand momentum transport for non-rotating initial state \( \Pi(\Omega_\varphi = 0) \)
  - \( \Pi(\Omega_\varphi = 0) > 0 \) expels momentum \( \partial \Omega_\varphi / \partial t < 0 \) (counter-current)
  - \( \Pi(\Omega_\varphi = 0) < 0 \) brings momentum \( \partial \Omega_\varphi / \partial t > 0 \) (co-current)
Symmetric turbulence cannot result in momentum transport

- Symmetry in gyrokinetic-Maxwell equations without formally small effects in rhostar $\rho_* = \rho_i / L_T$ [Sugama, PPCF (2011) and Parra, PoP (2011)]

$$\pi_0(\theta, v_\parallel, k_r) = -\pi_0(-\theta, -v_\parallel, -k_r)$$

$$\Pi_0(\Omega_\varphi = 0) = \sum_{k_r} \int d\theta d\Omega_\varphi \pi_0(\theta, v_\parallel, k_r) = 0$$
What are the perturbations that break the symmetry?

- Up-down asymmetric magnetic equilibrium [Camenen, PRL (2010) and Ball, MIT MS Thesis (2013)]

- Slow variation of radial profile (Global codes) [Wang, PoP (2010) and Waltz, PoP (2011)]

- Slow poloidal variation of turbulence [Barnes, PRL (2011) and Sung, PoP (2013)]

- Small deviation from Maxwellian equilibria (diamagnetic effect) [Lee, MIT Thesis (2013) and Barnes, PRL (2013)]

Except up-down asymmetry, all effects are small in \( \rho_\star = \rho_i / L_T \)
Can diamagnetic flows induce intrinsic rotation?

- Ion toroidal rotation in a tokamak is

$$V_\varphi = \Omega_\varphi R = (\Omega_\varphi, d + \Omega_\varphi, E) R$$

- For non-rotating state in which the diamagnetic flow and the ExB flow cancel each other, finite momentum flux occurs

$$\Omega_\varphi = \Omega_\varphi, d + \Omega_\varphi, E = 0 \quad \Rightarrow \quad \Pi(\Omega_\varphi = 0) \neq 0$$

diamagnetic ExB flow

$$\Omega_\varphi, d = -\frac{1}{B_\theta R} \frac{c}{Z e n_i} \left( \frac{\partial p_i}{\partial r} + C_T(\theta) \frac{\partial T_i}{\partial r} \right)$$
due to pressure and temperature gradient

$$\Omega_\varphi, E = \frac{c E_r}{R B_\theta}$$
due to radial electric field
The diamagnetic flow and the ExB flow occur due to different mechanisms.

- Radial displacement due to magnetic drifts depends on the sign of parallel velocity:
  \[ \Delta r \sim \left( \frac{B}{B_\theta} \right) \rho_i \]
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The diamagnetic flow and the ExB flow occur due to different mechanisms

- Radial displacement due to magnetic drifts depends on the sign of parallel velocity
  \[ \Delta r \sim (B/B_\theta)\rho_i \]

- Diamagnetic particle flow: hotter and denser inner plasmas result in larger particle flux

\[
\Omega_{\varphi, dR} \sim \Delta r \frac{\partial \ln \rho_i}{\partial r} v_{ti} \\
\sim \frac{B}{B_\theta L_P} \rho_i v_{ti}
\]
The diamagnetic flow and the ExB flow occur due to different mechanisms.

- Radial displacement due to magnetic drifts depends on the sign of parallel velocity:
  \[ \Delta r \sim \left( \frac{B}{B_\theta} \right) \rho_i \]

- ExB flow: radial electric field changes the particle energy, and the average parallel velocity.
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- Radial displacement due to magnetic drifts depends on the sign of parallel velocity:
  \[ \Delta r \sim \left( \frac{B}{B_\theta} \right) \rho_i \]

- ExB flow: radial electric field changes the particle energy, and the average parallel velocity:
  \[ mv_\varphi \Delta v_\varphi \sim Ze \Delta r \left( \frac{\partial \phi}{\partial r} \right) \]

\[ \Omega_{\varphi,E} R \sim \Delta v_\varphi \sim \frac{B}{B_\theta} \rho_i \frac{Ze}{T} \frac{\partial \phi}{\partial r} \nu_{ti} \]
Gyrokinetic equations with diamagnetic flow ($\Omega_{\varphi,d}$) and ExB flow ($\Omega_{\varphi,E}$)

- For low flow (Mach~ 0.1) without external torque,

$$\Omega_{\varphi,E} \sim \Omega_{\varphi,d} \sim \left(\frac{B}{B_\theta}\right)\left(\frac{\rho_i}{L_T}\right)v_{ti}$$
Gyrokinetic equations with diamagnetic flow ($\Omega_{\varphi,d}$) and ExB flow ($\Omega_{\varphi,E}$)

- For low flow (Mach\,$\sim\,$ 0.1) without external torque,

\[
\Omega_{\varphi,E} \sim \Omega_{\varphi,d} \sim (B/B_\theta)(\rho_i/L_T)v_{ti}
\]

- Lowest order gyrokinetic equation without any flow in the lab frame is

\[
\frac{\partial f^{tb}}{\partial t} + \left(v_{||} \hat{b} + \mathbf{v}_M + \mathbf{v}_E^{tb}\right) \cdot \nabla f^{tb}
\]

\[
= -\mathbf{v}_E^{tb} \cdot \nabla F_0 + \frac{Ze}{m_i} \left[v_{||} \hat{b} + \mathbf{v}_M\right] \cdot \nabla \langle \phi^{tb} \rangle \frac{\partial F_0}{\partial E} + \langle C(f) \rangle
\]
Gyrokinetic equations with diamagnetic flow ($\Omega_{\varphi,d}$) and ExB flow ($\Omega_{\varphi,E}$)

- For low flow ($\text{Mach} \approx 0.1$) without external torque,
  \[ \Omega_{\varphi,E} \sim \Omega_{\varphi,d} \sim \left(\frac{B}{B_\theta}\right)\left(\frac{\rho_i}{L_T}\right)v_{ti} \]

- Poloidal rhostar $\left(\frac{B}{B_\theta}\right)\left(\frac{\rho_i}{L_T}\right)$ higher order correction to gyrokinetic equation in the lab frame is
  \[
  \frac{\partial f^{tb}}{\partial t} + \left(v_{||}\hat{b} + v_M + (v_E + v_{E0})\right) \cdot \nabla f^{tb} - \frac{Ze}{m_i} v_M \cdot \nabla \phi_0 \frac{\partial f^{tb}}{\partial E} \\
  = -v_E^{tb} \cdot \nabla (F_0 + F_1) + \frac{Ze}{m_i} [v_{||}\hat{b} + v_M] \cdot \nabla \langle \phi^{tb} \rangle \frac{\partial (F_0 + F_1)}{\partial E} + \langle C(f) \rangle
  \]

where $F_0 = F_M (v - \Omega_{\varphi,E} R \hat{\varphi})$ is the shifted Maxwellian only by ExB flow,
Gyrokinetic equations with diamagnetic flow ($\Omega_{\varphi,d}$) and ExB flow ($\Omega_{\varphi,E}$)

- For low flow (Mach~ 0.1) without external torque,
  \[
  \Omega_{\varphi,E} \sim \Omega_{\varphi,d} \sim (B/B_\theta)(\rho_i/L_T)v_{ti}
  \]

- Poloidal rhostar \((B/B_\theta)(\rho_i/L_T)\) higher order correction to gyrokinetic equation in the lab frame is
  \[
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  \]
  \[
  = -v^{tb}_E \cdot \nabla (F_0 + F_1) + \frac{Ze}{m_i} [v_{||} \hat{b} + v_M] \cdot \nabla \langle \phi^{tb} \rangle \frac{\partial (F_0 + F_1)}{\partial E} + \langle C(f) \rangle
  \]

  where \( F_0 = F_M (v - \Omega_{\varphi,E} R\hat{\varphi}) \) is the shifted Maxwellian only by ExB flow,
Gyrokinetic equations with diamagnetic flow \((\Omega_{\varphi,d})\) and ExB flow \((\Omega_{\varphi,E})\)

- For low flow (Mach~ 0.1) without external torque,
  \[
  \Omega_{\varphi,E} \sim \Omega_{\varphi,d} \sim \left(\frac{B}{B_\theta}\right)\left(\frac{\rho_i}{L_T}\right)v_{ti}
  \]

- Poloidal rhostar \((\frac{B}{B_\theta})\left(\frac{\rho_i}{L_T}\right)\) higher order correction to gyrokinetic equation in the lab frame is

\[
\frac{\partial f^{tb}}{\partial t} + \left( v_{\parallel} \hat{b} + v_M + (v_{E0} + v_{E}^{tb}) \right) \cdot \nabla f^{tb} - \frac{Ze}{m_i} v_M \cdot \nabla \phi_0 \frac{\partial f^{tb}}{\partial E} = -v_{E}^{tb} \cdot \nabla (F_0 + F_1) + \frac{Ze}{m_i} [v_{\parallel} \hat{b} + v_M] \cdot \nabla \langle \phi^{tb} \rangle \frac{\partial (F_0 + F_1)}{\partial E} + \langle C(f) \rangle
\]

where \(F_0 = F_M(v - \Omega_{\varphi,E} R \hat{\varphi})\) is the shifted Maxwellian only by ExB flow,

and \(F_1\) includes the diamagnetic flow

\[
F_1 = \frac{B_\varphi}{B} \frac{m_i v_{\parallel} \Omega_{\varphi,d} R}{T_i} F_M + ...
\]
Correction to Maxwellian equilibria via neoclassical distribution function

- Neoclassical distribution function is obtained by drift-kinetic equations

\[ F_1 \left( \frac{\partial p}{\partial r}, \frac{\partial T}{\partial r}, q, \nu_\star, \ldots \right) = F_1^{V\parallel} + F_1^{other} \approx \frac{B}{B_\theta L_T} F_M \]

- Parallel particle flow piece:

\[ F_1^{V\parallel} = \frac{B_\varphi}{B} \frac{m_i v_{\parallel} \Omega_\varphi, dR}{T_i} F_M \]

- Other pieces are due to the higher moments in \( v_{\parallel} \) (e.g. parallel heat flow)
Different transport for diamagnetic flow and ExB flow results in intrinsic momentum transport

- For both diamagnetic flow and ExB flow, \( \Omega_\varphi = \Omega_\varphi, d + \Omega_\varphi, E \)

\[
\Pi \left( \Omega_\varphi, d, \Omega_\varphi, E, \frac{\partial \Omega_\varphi, d}{\partial r}, \frac{\partial \Omega_\varphi, E}{\partial r}, \ldots \right)
\]
Different transport for diamagnetic flow and ExB flow results in intrinsic momentum transport

- For both diamagnetic flow and ExB flow, \( \Omega_\varphi = \Omega_\varphi, d + \Omega_\varphi, E \)

\[
\Pi \left( \Omega_\varphi, d, \Omega_\varphi, E, \frac{\partial \Omega_\varphi, d}{\partial r}, \frac{\partial \Omega_\varphi, E}{\partial r}, \ldots \right)
\]

\[
\simeq \Pi_{other} - n_i m_i R^2 \left[ P_{\varphi, d} \Omega_{\varphi, d} + P_{\varphi, E} \Omega_{\varphi, E} \right] - n_i m_i R^2 \left[ \chi_{\varphi, d} \frac{\partial \Omega_{\varphi, d}}{\partial r} + \chi_{\varphi, E} \frac{\partial \Omega_{\varphi, E}}{\partial r} \right]
\]
Different transport for diamagnetic flow and ExB flow results in intrinsic momentum transport

- For both diamagnetic flow and ExB flow, \( \Omega_\varphi = \Omega_{\varphi,d} + \Omega_{\varphi,E} \)

\[
\Pi \left( \Omega_{\varphi,d}, \Omega_{\varphi,E}, \frac{\partial \Omega_{\varphi,d}}{\partial r}, \frac{\partial \Omega_{\varphi,E}}{\partial r}, \ldots \right) 
\approx \Pi_{other} - n_im_iR^2 \left[ P_{\varphi,d} \Omega_{\varphi,d} + P_{\varphi,E} \Omega_{\varphi,E} \right] - n_im_iR^2 \left[ \chi_{\varphi,d} \frac{\partial \Omega_{\varphi,d}}{\partial r} + \chi_{\varphi,E} \frac{\partial \Omega_{\varphi,E}}{\partial r} \right]
\]

- For the diamagnetic flow and ExB flow canceling each other,

\( \Omega_\varphi = \Omega_{\varphi,d} + \Omega_{\varphi,E} = 0 \) and \( \frac{\partial \Omega_\varphi}{\partial r} = \frac{\partial \Omega_{\varphi,d}}{\partial r} + \frac{\partial \Omega_{\varphi,E}}{\partial r} = 0 \)
Different transport for diamagnetic flow and ExB flow results in intrinsic momentum transport

- For both diamagnetic flow and ExB flow, 
  \[ \Omega_\varphi = \Omega_\varphi,d + \Omega_\varphi,E \]

\[
\Pi \left( \Omega_\varphi,d, \Omega_\varphi,E, \frac{\partial \Omega_\varphi,d}{\partial r}, \frac{\partial \Omega_\varphi,E}{\partial r}, \ldots \right) \]

\[ \simeq \Pi_{other} - n_im_iR^2 \left[ P_\varphi,d \Omega_\varphi,d + P_\varphi,E \Omega_\varphi,E \right] - n_im_iR^2 \left[ \chi_\varphi,d \frac{\partial \Omega_\varphi,d}{\partial r} + \chi_\varphi,E \frac{\partial \Omega_\varphi,E}{\partial r} \right] \]

- For the diamagnetic flow and ExB flow canceling each other,
  \[ \Omega_\varphi = \Omega_\varphi,d + \Omega_\varphi,E = 0 \quad \text{and} \quad \frac{\partial \Omega_\varphi}{\partial r} = \frac{\partial \Omega_\varphi,d}{\partial r} + \frac{\partial \Omega_\varphi,E}{\partial r} = 0 \]

\[ \Pi_{int} = \Pi_{other} - n_im_iR^2 \Delta P_\varphi \Omega_\varphi,d - n_im_iR^2 \Delta \chi_\varphi \left( \frac{\partial \Omega_\varphi,d}{\partial r} \right) \]

where \( \Delta P_\varphi = P_\varphi,d - P_\varphi,E \) and \( \Delta \chi_\varphi = \chi_\varphi,d - \chi_\varphi,E \)
Different transport for diamagnetic flow and ExB flow results in intrinsic momentum transport

- For both diamagnetic flow and ExB flow, \( \Omega_\varphi = \Omega_\varphi,d + \Omega_\varphi,E \)

\[
\Pi \left( \Omega_\varphi,d, \Omega_\varphi,E, \frac{\partial \Omega_\varphi,d}{\partial r}, \frac{\partial \Omega_\varphi,E}{\partial r}, \cdots \right)
\approx \Pi_{other} - n_i m_i R^2 \left[ P_{\varphi,d} \Omega_\varphi,d + P_{\varphi,E} \Omega_\varphi,E \right] - n_i m_i R^2 \left[ \chi_\varphi,d \frac{\partial \Omega_\varphi,d}{\partial r} + \chi_\varphi,E \frac{\partial \Omega_\varphi,E}{\partial r} \right]
\]

- For the diamagnetic flow and ExB flow canceling each other,

\[
\Omega_\varphi = \Omega_\varphi,d + \Omega_\varphi,E = 0 \quad \text{and} \quad \frac{\partial \Omega_\varphi}{\partial r} = \frac{\partial \Omega_\varphi,d}{\partial r} + \frac{\partial \Omega_\varphi,E}{\partial r} = 0
\]

\[
\Pi_{\text{int}} = \Pi_{other} - n_i m_i R^2 \Delta P_\varphi \Omega_\varphi,d - n_i m_i R^2 \Delta \chi_\varphi \left( \frac{\partial \Omega_\varphi,d}{\partial r} \right)
\]

\[
\Pi_{\text{int}} \left( \Omega_\varphi,d, \frac{\partial \Omega_\varphi,d}{\partial r}, \Pi_{other} \right) \rightarrow \Pi_{\text{int}} \left( \frac{\partial p_i}{\partial r}, \frac{\partial T_i}{\partial r}, q, \nu_*, \frac{\partial^2 p_i}{\partial r^2}, \frac{\partial^2 T_i}{\partial r^2}, \frac{\partial q}{\partial r}, \frac{\partial \nu_*}{\partial r} \right)
\]

Example 3  Example 1  Example 2
Example 1: Reversal of intrinsic rotation direction

< Observed reversal at Alcator C-Mod>

- Low n & high $I_p$: Co-rotation
- High n & low $I_p$: Counter-rotation

Rice, NF(2011)
Example 1: Reversal of intrinsic rotation direction

< Observed reversal at Alcator C-Mod>

- Low n & high I_p: Co-rotation
- High n & low I_p: Counter-rotation

< Estimation of intrinsic momentum flux>

- ExB flow cancels with diamagnetic flow
- From negative flux to positive flux at $\nu_\ast \sim 1$ (from Banana to Plateau)

Rice, NF(2011)

Simulated for Cyclone base case using GS2 and $F_1$ from NEO [Barnes (2013)]
Example 2: Saturation of the rotation change in the lower hybrid wave momentum injection

< Observed rotation at Alcator C-Mod>

Rice, NF(2013)
Example 2: Saturation of the rotation change in the lower hybrid wave momentum injection

< Observed rotation at Alcator C-Mod>

- Acceleration in counter-current direction by the wave momentum
Example 2: Saturation of the rotation change in the lower hybrid wave momentum injection

< Observed rotation at Alcator C-Mod >

- Acceleration in counter-current direction by the wave momentum
- Different steady state rotation (High current vs. Low current)
Example 2: Saturation of the rotation change in the lower hybrid wave momentum injection

< Observed rotation at Alcator C-Mod>

- Acceleration in counter-current direction by the wave momentum
- Different steady state rotation (High current vs. Low current)

< Estimation of intrinsic momentum flux>
- Decrease of magnetic shear
  → Reduced positive momentum flux
  → Reduced counter-current rotation

\[
\hat{s} = \left(\frac{r}{q}\right)(dq/dr)
\]

- For low $I_p$, LHCD changes q profile significantly in a resistive time $O(100)$ msec
Example 3: Peaked profile of intrinsic rotation in L-H transition

- Observed increase of the co-current core rotation in the transition from L-mode to H-mode

- Scaling of the intrinsic flow:

\[ \Delta V_\varphi = C_v \Delta W/I_p \]

For Alcator C-Mod

\[ C_v \approx 700 [km/s] [A/J] \]

Rice, NF (2007)
Example 3: Larger momentum pinch of diamagnetic flow may cause the rotation peaking

• Strong co-current diamagnetic flow and counter-current $\text{ExB}$ flow at pedestal (e.g. $\Delta T_i \sim -1\,\text{keV}, \Delta r_p \sim 1\,\text{cm}, B_\theta \sim 0.5T \Rightarrow \Omega_{\varphi, d} R_0 \sim 200\,\text{km/s} \sim -\Omega_{\varphi, E} R_0$)
Example 3: Larger momentum pinch of diamagnetic flow may cause the rotation peaking

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- Negative intrinsic momentum transport $\Pi_{\text{int}}^\Delta P_{\varphi} < 0$ due to $P_{\varphi,d} > P_{\varphi,E}$

$$\Pi_{\text{int}}^\Delta P_{\varphi} = -n_i m_i (P_{\varphi,d} - P_{\varphi,E}) \langle \Omega_{\varphi,d} R_0^2 \rangle_s$$

$$P_{\varphi,E}/\chi_{\varphi} \simeq 2.9/R_0$$

$$P_{\varphi,d}/\chi_{\varphi} \simeq 3.5/R_0$$

Simulation parameters:
R/L_T=9.0, R/L_n=9.0,
r/a=0.8, q=2.5
Example 3: Larger momentum pinch of diamagnetic flow may cause the rotation peaking

- Strong co-current diamagnetic flow and counter-current ExB flow at pedestal (e.g. $\Delta T_i \sim -1keV, \Delta r_p \sim 1cm, B_\theta \sim 0.5T \Rightarrow \Omega_{\varphi,d} R_0 \sim 200km/s \sim -\Omega_{\varphi,E} R_0$)

- Negative intrinsic momentum transport $\Pi_{\text{int}}^{\Delta P_{\varphi}} < 0$ due to $P_{\varphi,d} > P_{\varphi,E}$

- Estimation of the core rotation peaking

\[
\Delta V_{\varphi} \propto \Delta r_p \Pi_{\text{int}}^{\Delta P_{\varphi}} \\
\propto \Delta r_p \Delta P_{\varphi} \Omega_{\varphi,d} \\
= C_v \Delta W / I_p
\]

For Alcator C-Mod,

$C_v \approx 630[km/s][A/J]$
Conclusions

• Inclusion of diamagnetic correction to equilibrium Maxwellian distribution function results in intrinsic momentum transport for non-rotating state (i.e. $\Omega_\varphi = \Omega_\varphi,d + \Omega_\varphi,E = 0$)

• Collisional dependence of the intrinsic momentum transport is qualitatively consistent with experimentally observed intrinsic rotation reversal

• The feature of intrinsic momentum flux depending on the magnetic shear may be helpful to explain the different rotation change depending on the plasma current in the lower hybrid wave injection.

• Larger momentum pinch for diamagnetic flow than for ExB flow may explain the peaked profile of intrinsic rotation in H-mode
Appendix
Example 3: Saturation of the rotation change due to the lower hybrid wave momentum injection

- Acceleration in counter-current direction by the wave momentum
- Different steady state rotation (High current vs. Low current)

### Table: 

<table>
<thead>
<tr>
<th>Condition</th>
<th>Before LH Wave</th>
<th>After O(10) msec</th>
<th>After 400 msec</th>
</tr>
</thead>
<tbody>
<tr>
<td>High $I_p$</td>
<td>-20 km/s</td>
<td>-50 km/s</td>
<td></td>
</tr>
<tr>
<td>Low $I_p$</td>
<td>-40 km/s</td>
<td>-20 km/s</td>
<td></td>
</tr>
</tbody>
</table>

Measured using X-ray crystal spectrometer in Alcator C-Mod by Y. Podpaly, J. Rice, M. Reinke, et al.
Example 2: Saturation of the rotation change in the lower hybrid wave momentum injection

< Observed rotation at Alcator C-Mod>

<table>
<thead>
<tr>
<th></th>
<th>Before LH wave turns on</th>
<th>After O(10) msec</th>
<th>After ~100 msec</th>
<th>Saturation after 400 msec</th>
</tr>
</thead>
<tbody>
<tr>
<td>High $I_p$</td>
<td>~700kA</td>
<td>-20 km/s</td>
<td>$\downarrow$</td>
<td>-50 km/s</td>
</tr>
<tr>
<td>Low $I_p$</td>
<td>~350kA</td>
<td>-40 km/s</td>
<td>$\downarrow$</td>
<td>-20 km/s</td>
</tr>
</tbody>
</table>

- Acceleration in counter-current direction by the wave momentum
- Different steady state rotation (High current vs. Low current)

< Estimation of intrinsic momentum flux>
- Decrease of magnetic shear
  → Reduced negative momentum flux
  → Reduced counter-current rotation

$$\hat{s} = \left(\frac{r}{q}\right) \left(\frac{dq}{dr}\right)$$

For low $I_p$, LHCD changes q profile significantly in a resistive time O(100) msec
The turbulent radial momentum transport can be linearized and split into momentum pinch, diffusion and intrinsic momentum flux.

- The momentum redistribution is due to the turbulent radial flux of toroidal angular momentum

\[
\Pi \simeq \left\langle m_i \int d^3 v f_i^{tb} (v \cdot \hat{\phi} R) (v_E^{tb} \cdot \hat{r}) \right\rangle_s
\]

- Each contribution of flow and radial flow shear to the turbulent radial flux can be linearized for low flow

\[
\Pi \left( \Omega_\varphi, \frac{\partial \Omega_\varphi}{\partial r}, \ldots \right) \simeq \Pi_{\text{int}} - P_\varphi n_i m_i R^2 \Omega_\varphi - \chi_\varphi n_i m_i R^2 \frac{\partial \Omega_\varphi}{\partial r}
\]

\[\begin{align*}
\text{pinch} & \quad \text{diffusion}
\end{align*}\]
Conclusions

Steady state rotation

Momentum redistribution

External Torque

Plasma current radial profile changes due to LH wave current drive

• Safety factor increase generally, and magnetic shear changes significantly depending on the location of LH wave power absorption.

• The change in the current profile due to the lower hybrid waves is in a current resistive relaxation time scale $O(100)$ msec, in which the reversal of the rotation change is observed.