Comparison of Edge Turbulence Velocity Analysis Techniques Using Gas Puff Imaging Data on Alcator C-Mod

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Abstract

- Two methods for analyzing turbulence velocities used in the past on Gas Puff Imaging (GPI) data
  - One based on time-delay cross-correlation of successive images that is used to track motion of discrete structures.
  - Other uses Fourier analysis to obtain frequency and poloidal wavenumber spectra, from which phase velocities are derived.
- Recent experiments completed with imaging at outboard midplane.
- Cross-correlation technique yielded magnitudes of poloidal velocities in the 0.1-1.4 km/s range.
- Fourier analysis yielded values up to an order of magnitude larger for the same data.
- To understand the reasons for these differences, we have created and analyzed synthetic data.
- Comparisons between the two analysis techniques applied to both the actual experimental and synthetic data will be presented.
Gas Puff Imaging on Alcator C-Mod

- Neutral gas puffed into plasma
- Puff region imaged onto fast-framing Phantom Camera
- Filtered for He I or D-α emission
- View Covers ~6x6 cm
- 64x64 pixels
- Typical frame rate is 391,000 frames/sec
Want to study edge turbulence and measure relevant quantities, such as phase velocities

Also want to be able to analyze Phantom Camera data in a couple of different ways
- Fourier Analysis technique (as developed by I. Cziegler\textsuperscript{2})
- Time Delay Estimation using successive images from the Phantom Camera (as developed by S. Zweben\textsuperscript{1}; ‘\textit{Z-TDE}’)

However, these two methods can disagree by up to an order of magnitude, and even direction for the same GPI data
Analysis of Poloidal Velocities of Structures in GPI Data for C-Mod Shots Yield Significantly Different Results

Z-TDE points from S. J. Zweben et al., PoP 20, 072503 (2013)
Analysis of Poloidal Velocities of Structures in GPI Data for C-Mod Shots Yield Significantly Different Results

Z-TDE points from S. J. Zweben et al., PoP 20, 072503 (2013)
Fourier Analysis

- Fourier transform signal from each pixel **temporally** to obtain frequency $f$
- Also Fourier transform **spatially** (over columns, in $z$-direction only) to obtain wavenumber $k$
- K-f spectrum used to give poloidal phase velocity of emission structures:
  \[ V_{phase} = \frac{2\pi f}{k} \]
- Different ways to obtain kf-spectrum give similar results (FFT$^2$, two-point$^3$)
Conditional \((k_{pol}, f)\) spectrum constructed by normalizing the spectral density at each frequency

Thus larger, slower features with typically more power are not favored over smaller and faster features

Conditional spectral density given by:\(^3\)

\[
s(k|\omega) = \frac{S(k, \omega)}{S(\omega)}
\]
Time Delay Estimation with Images

- As developed by S. Zweben (Z-TDE)
- Searches for max cross-correlation within a ±8 pixel search box, and with lags up to ±10 frames (±0.7cm and 25μs)

S. Zweben et al. APS 2012
Z-TDE with Images

- Averages over velocities from time lags with max cross-correlations (other than 0 lag) that are greater than 0.5
- Max detectable velocity = ±2.8 km/s

Max cross correlation vs lag over all search boxes and pixels

Velocity Map by pixel with separatrix and rho = 0.5, 1.0, 1.5, and 2.0 shown
Currently, neither code can be used to validate the other.

This demonstrates the need for a sure bet: the ability to input a blob field in which we know the exact radial and poloidal velocities beforehand.

Our approach is to develop synthetic blob fields to test each code rigorously, and show that those synthetic blob fields are as similar as possible to the real data.
Generation of Synthetic Data

- User can define blob size, distribution (normal or gamma)$^4$, intensity, velocity, and # of frames
- Choice of single or multiple blob fields

Synthetic blob field, moving completely vertically
Comparison of Real and Synthetic Data

Real
Shot 1120815021

Synthetic
Sample Synthetic Signal

Time Series

PDF
Changing Distribution Does Not Affect Velocity

- Selected synthetic blob maximum intensities from different gamma and normal distributions as shown
Trials were completed with synthetic data as shown.

Corrected by assuming blob motion can be represented by a wavefront.

Fourier: $V_{pol}$ (Output) is Artificially Large when $V_{rad}$ Input is Large.
Structures Move Primarily Vertically for Most Shots

- Please see movie of shot 1120712026 playing on laptop
- Thus correction to the Fourier velocities is likely < 2
- Velocity maps also indicate mostly vertical motion
**Z-TDE: \( V_{pol} \) and \( V_{rad} \) (Output)**

Underestimated

- Trials with synthetic data completed by varying \( V_Z \) while keeping \( V_R \) constant (left)
- Trials with synthetic data completed by varying \( V_R \) while keeping \( V_Z \) constant (right)
Systematic Issues Still Do Not Account for Total Difference in Velocities

Maximum change factor of 1.2 based on velocity map for the Fourier analysis and correction using a linear fit for the Z-TDE analysis show that the systematic issues do not account for total discrepancy.
Systematic Issues Still Do Not Account for Total Difference in Velocities

Maximum change factor of 1.2 to 4 based on velocity map for the Fourier analysis and correction using a linear fit for the Z-TDE analysis show that the systematic issues do not account for total discrepancy.
Noisy time signals can lead to ambiguities in finding the maximum cross-correlation and thus, large error bars on the velocities.

The different analyses may weight different sizes, intensities, and speeds differently.

Possible non-linear dispersion ($\omega/k$ is not constant over all $k$).

Changing background signal (e.g. slow increase due to gas puff, etc.).
Two Ways to Obtain Large Error Bars from Cross-Correlations

• May be unable to find the correct peak if multiple time lags have similar values (e.g. sine wave due to waves in the plasma)

• Also possible that cross-correlation versus lag time has a shallow peak

Illustrations of possible issues
Real Data Show Shallow Peaks Only for Cross-Correlations Below Threshold

Max cross correlations have defined peaks at 0 lag.

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Shot 1120712026

Shot 1120224027

Shot 1120712026

Shot 1120224027

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Threshold = 0.5
Z-TDE Tends to Favor Slower-Moving Blobs

Summary of three trial shots with two different blob fields:

<table>
<thead>
<tr>
<th>Input $V_{\text{rad}}$ (km/s)</th>
<th>Input $V_{\text{pol}}$ (km/s)</th>
<th>FWHM (cm)</th>
<th>Intensity</th>
<th>Output $V_{\text{rad}}$</th>
<th>Output $V_{\text{pol}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>-0.14</td>
<td>2.34</td>
<td>0.4</td>
<td>2.0</td>
<td>0.04</td>
<td>0.38</td>
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<tr>
<td>0.01</td>
<td>0.44</td>
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<td>3.0</td>
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<td></td>
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<tr>
<td>0.14</td>
<td>1.60</td>
<td>0.8</td>
<td>3.0</td>
<td>0.036</td>
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<td>0.02</td>
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<td>0.8</td>
<td>3.0</td>
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<td>0.14</td>
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<td>0.8</td>
<td>3.5</td>
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<td>0.40</td>
<td>0.4</td>
<td>2.5</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Fourier Better at Distinguishing Different Sizes and Intensities

Predicted: 2.36 km/s and 0.44 km/s
Predicted: 1.66 km/s and 0.41 km/s
Predicted: 1.66 km/s and 0.41 km/s

As long as the FWHM of the structures/blobs is at least 2 pixels (Nyquist sampling), blob size does not matter.
## Summary of Tests

<table>
<thead>
<tr>
<th>Testing for:</th>
<th>Fourier Analysis</th>
<th>Z-TDE Analysis</th>
</tr>
</thead>
<tbody>
<tr>
<td>Systematic Errors</td>
<td>✔️</td>
<td>✔️</td>
</tr>
<tr>
<td>Ambiguous Cross-Correlation</td>
<td>Not Applicable</td>
<td>✔️</td>
</tr>
<tr>
<td>Dependence of Output on Blob Size/Intensity</td>
<td>❌</td>
<td>❌</td>
</tr>
<tr>
<td>How Each Method Weights Blob Speed</td>
<td>❌</td>
<td>❌</td>
</tr>
<tr>
<td>Non-linear Dispersion</td>
<td>✗</td>
<td>✗</td>
</tr>
<tr>
<td>Changing background signal</td>
<td>✔️</td>
<td>❌</td>
</tr>
</tbody>
</table>

- ✔️ Completed
- ❌ Not yet completed
- ❌ More tests needed
Future Work

• Further investigate whether or not the methods favor certain blob intensities, sizes, or speeds
• Investigate data for possible non-linear dispersion
• Investigate data for possible changes in background
• Study radial cross-correlations lengths to indicate the magnitude of radial motion in the real data, currently assumed small
• Analyze both synthetic and real data with different TDE technique
Two methods (Fourier and Z-TDE) for obtaining velocities of structures in GPI data disagree by up to an order of magnitude and sometimes direction.

Created synthetic images to investigate and test each method rigorously.

Synthetic images have some of the statistical properties of the real data.

Found systematic issues and can correct for some of them.

Begun to eliminate other possible explanations.
Acknowledgements and References

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1 S. J. Zweben et al., PoP 20, 072503 (2013)

2 Cziegler, I. Turbulence and Transport Phenomena in Edge and Scrape-Off-Layer Plasmas (Doctoral Dissertation), MIT (2011)


4 O. E. Garcia et al., JNuM 438, S180 (2013)
Backup Slides
Fourier: Two Lobes in Shot 1120224009

- IDD vel = -2.45 km/s
- EDD vel = 2.11 km/s
- $\rho = 0.65 \pm 0.23$ cm
- $R_{av} = 89.90$ cm
Different Methods of Obtaining KF-Spectrum Give Same Result

Input: blob field moving vertically at 1.01 km/s
Wavefront Model Provides Correction

- Column transform gives
  \[ \lambda_z = \frac{\lambda}{\sin \alpha}, \quad k_z = \frac{2\pi}{\lambda_z} \]
- Similarly, row transform gives
  \[ \lambda_R = \frac{\lambda}{\cos \alpha}, \quad k_R = \frac{2\pi}{\lambda_R} \]
- Since phase velocity is \( \frac{2\pi f}{k} \), this gives:
  \[ V_z = f \lambda_z = \frac{f \lambda}{\sin \alpha} = \frac{V}{\sin \alpha} \]
  \[ V_R = \frac{V}{\cos \alpha} \]
Trials Indicate Model is a Good Fit

After plugging everything in:

\[
\frac{V'_z}{V_z} = 1 + \left(\frac{V_R}{V_z}\right)^2
\]

\[
\frac{V'_R}{V_R} = 1 + \left(\frac{V_z}{V_R}\right)^2
\]