Scalings of nonlinearity in the edge plasma and its connection to transition thresholds

I. Cziegler, P. H. Diamond, G. R. Tynan
UCSD, Center for Momentum Transport and Flow Organization

A. E. Hubbard, J. W. Hughes, J. L. Terry, J. H. Irby
MIT Plasma Science and Fusion Center

UC San Diego

Alcator C-Mod
Nonlinear transfer processes between large-scale edge flows and the ambient broadband fluctuations have been shown to play a significant role in the dynamics of edge turbulence, including spreading power from coherent modes and suppressing turbulence at the formation of edge transport barriers. In order to predict thresholds of confinement regimes, both the transition dynamics and the scalings of the nonlinear energy transfer must be studied. Since the expected flow damping terms depend on ion collision rates and local safety factor, recent experiments aimed also to explore the nonlinear drive at various values of the plasma current, density and amount of auxiliary heating. Nonlinear interactions between zonal flows in L-mode and geodesic-acoustic modes (GAM) in I-mode plasmas are estimated using bispectral as well as time-resolved methods based on gas-puff-imaging.

Supported by USDoE Award DE-FC02-99ER54512
Motivations and Background
The importance of the L-H trigger

The L-H transition is widely reproduced in fusion devices. H-mode forms the baseline operation scenario for ITER and further tokamaks.

The threshold of net heating power necessary for access is currently not predicted. Even the regression error of extrapolations for future fusion devices covers an order of magnitude.

**Need a physics based, quantitative model for predicting transition and understanding alternative high confinement regimes**

First step: focus on immediate trigger of L-H transition

5-10ms around L-H transition

...in the region where the edge transport barrier forms.
Eventually, the L-H transition might need to be avoided rather than achieved:

- Edge heat and mass transport channels are closely coupled - “pedestal”
- Enhanced confinement of particles means **impurities can accumulate**
- Gradients hit stability limits (peeling-ballooning): **ELM crashes lead to prohibitive rate of wall erosion**
- **Confinement degrades** with increasing heating power: $W_p \sim P^{0.3}$

**I-mode:**

- Decouples energy and particle transport, with H-mode energy- and L-mode mass-confinement, flushing impurities from plasma continuously.
- ELM-suppressed, avoiding damage to wall/divertor without active ELM control.
- Energy confinement degrades weakly or not at all with heating power - scales favorably to reactor scale devices
H-mode unfavorable grad-B direction (away from active X-point) favors access to regime.

Only regime in Alcator C-Mod with poloidal velocities exhibiting geodesic acoustic modes (GAM).

Mode presence correlated with I-mode characteristic weakly coherent mode (WCM).

Not clear which mode (if any) is responsible for unique quality of transport.
GAM shown to be responsible for broad frequency range of WCM via nonlinear coupling

The weakly coherent mode is predominantly a density fluctuation. Consequently, the relevant nonlinear transfer process is the flow mediated three wave coupling between density fluctuations.

\[ T_n(f_1, f) = - \Re \langle \tilde{n}_f \tilde{u}_{f_1} \partial_i \tilde{n}_{f_1} \rangle \]

nonlinear transfer term
Diagnostic Setup

Primary diagnostic: **Gas Puff Imaging**
- gas puff injects neutral $D_2$, or He, sensitive to $n_e$, $T_e$
- small toroidal extent (~5 cm) allows localization
- outboard 90 channels cover ~ 4 cm x 3.6 cm
- views coupled to avalanche photodiodes (APD), sampled at 2 MHz

Temperature and density profiles: **Electron-Cyclotron Emission**
- grating polychromator set with radial views (resolution ~1 cm)
- time resolution at sub-millisecond scale

**Fast Two-Color Interferometer**
- line integrated density measurement
- time resolved at 0.1 ms

**Thomson Scattering**
- $n_e$ resolved in minor radius, mapped to $z = 0$
- time resolved at 10 ms
Averages for the analysis are calculated from vertical array of views.

Averaging in Reynolds decomposition in the model, \( \langle \nu \rangle \) also involves a separation of time scales, such that effectively

\[
\langle \nu \rangle = \langle \nu \rangle_{\theta,t}
\]

Scale separation is realized by digital filtering. The particular cutoff frequencies do not significantly affect the final result.

Separation between zonal flow and turbulence scales is chosen based on GAM studies as half the slowest observed GAM frequency. Turbulence filtered to effective Nyquist frequency,

\[
\tilde{\nu} = \nu|_{f=5kHz}^{50kHz}
\]

with a 10\( \mu \)s sample size per point.
Motivation of time-resolved analysis technique

\[ \partial_t \nu_\theta + v_r \partial_r \nu_\theta = \mu \partial_r^2 \nu_\theta, \]

Momentum equation for incompressible fluid

\[ v = \langle v \rangle + \tilde{v}, \quad \langle \tilde{v} \rangle = 0 \]

Reynolds decomposition

\[ \frac{1}{2} \partial_t \langle \tilde{v}_\theta^2 \rangle = -\langle \tilde{v}_r \tilde{v}_\theta \rangle \partial_r \langle \nu_\theta \rangle - \frac{1}{2} \partial_r \langle \tilde{v}_r \tilde{v}_\theta^2 \rangle + \mu \langle \tilde{v}_\theta \partial_r^2 \tilde{v}_\theta \rangle \]

kinetic energy in turbulence

\[ \frac{1}{2} \partial_t \langle \nu_\theta \rangle^2 = -\langle \nu_\theta \rangle \partial_r \langle \tilde{v}_r \tilde{v}_\theta \rangle + \mu \langle \nu_\theta \rangle \partial_r^2 \langle \nu_\theta \rangle \]

\[ = \langle \tilde{v}_r \tilde{v}_\theta \rangle \partial_r \langle \nu_\theta \rangle - \partial_r [ \langle \tilde{v}_r \tilde{v}_\theta \rangle \langle \nu_\theta \rangle ] + \mu \langle \nu_\theta \rangle \partial_r^2 \langle \nu_\theta \rangle \]

kinetic energy in low freq flow

Reynolds stress mediated zonal flow production:

\[ P = \langle \tilde{v}_r \tilde{v}_\theta \rangle \partial_r \langle \nu_\theta \rangle \]

Transport terms in the form of energy flux:

\[ \tilde{T} = \frac{\langle \tilde{v}_r \tilde{v}_\theta^2 \rangle}{2} \quad \tilde{\tilde{T}} = \langle \tilde{v}_r \tilde{v}_\theta \rangle \langle \nu_\theta \rangle \]

Only term missing is the turbulence drive from free energy of the back-ground gradients. Insert ad hoc drive term, and simplify rates to get:

\[ \partial_t \tilde{K} = \gamma_{\text{eff}} \tilde{K} - P - \partial_r \tilde{T} \quad \text{with } \gamma_{\text{eff}}: \text{net difference of drive and decorrelation} \]

\[ \partial_t \tilde{K} = P - \partial_r \tilde{T} - \nu_{\text{LF}} \tilde{K} \]
Velocity profiles constructed from GPI velocimetry and diamagnetic flows from temperature and density profiles

Direct GPI measurements and fluid velocity estimates, 0.5 ms during L-mode and during L-H transition.

The largest poloidal velocity deviation during the L-H transition is observed in the radial region at the channels GPI1 and GPI2. This location also exhibits the largest radial velocity shear.
Sample traces

Low frequency flows: from 3 separate experiments

<table>
<thead>
<tr>
<th>Conditions</th>
<th>Time History</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bt=5.2T Ip=0.8MA P=2.1MW</td>
<td></td>
</tr>
<tr>
<td>Bt=4.0T Ip=0.8MA P=1.5MW</td>
<td></td>
</tr>
<tr>
<td>Bt=5.4-2.8T Ip=0.8MA Ohmic</td>
<td></td>
</tr>
</tbody>
</table>

Poloidal velocity excursion is reproduced in all H-mode favorable transitions regardless of
- the kind of L-H transition
- magnetic field
- threshold power
- geometry

Time histories are aligned by Dα light drop at t = 0ms

Turbulent velocities:
Trigger of the L-H transition
Critical parameters

First order condition on turbulence collapse: terms taking energy from turbulence into zonal flows must exceed the drive. Written with symbols from the model equations:

\[ R_T \equiv \frac{P + \partial_r \tilde{T}}{\gamma_{\text{eff}} \tilde{K}} > 1, \quad 1\text{D including the energy-flux-like term} \]

where transfer terms and the kinetic energy are directly calculated, and \( \gamma_{\text{eff}} \) can be estimated from the model Eq. in a stationary L-mode as

\[
\nu_{\text{LF}}|_L = \left[ \frac{\partial_r \langle \tilde{v}_r \tilde{v}_\theta \rangle}{\langle v_\theta \rangle} \right]_{\text{L-mode}}
\]

\[
\gamma_{\text{eff}}|_L = \left[ \frac{P + \partial_r \tilde{T}}{\tilde{K}} \right]_{\text{L-mode}}
\]

Full L-H transition requires growth of non-turbulence driven shear, ie pressure gradient, to lock in H-mode once turbulence is reduced.

Results on Nonlinear Kinetic Energy Transfer
Results unambiguously establish the L-H transition sequence as:

1) Turbulence transiently increases (in the case shown due to heat pulse from sawtooth crash)

2) Normalized prod. term $R_T$ exceeds 1

3) Turbulence transfers energy to flow

4) Full pedestal evolves

5) $\nabla p$ locks in the H-mode

- shaded area: total transferred kinetic energy from turbulence to the low frequency Zonal Flow

- pedestal pressure gradient increases by about 100% during the period of significant nonlinear transfer
Result confirms that the experimental value of work done by the Reynolds stress is adequate for driving the local flow. Radial transport of produced ZF power is not negligible!
For a quantitative test of the adequacy of the mechanism to cause the full extent of turbulence quenching, integrate the model Eq. Then, ideally:

\[ \ln \frac{\tilde{K}_L}{\tilde{K}_{\text{min}}} = \int_{t_0}^{t_d} \left[ \frac{P + \partial_r \tilde{T}}{\tilde{K}} - \gamma_{\text{eff}} \right] dt, \]

- For consistency, the turbulence reduction logarithm must not be greater than the total power lost to ZF.
- Nonlinear energy transfer exceeds the amount that is expected if this is the dominant turbulence quenching mechanism.
- 0D, large error bars, can be improved if full radial structure is considered.

In 0D, kinetic energy transfer is larger than turbulence reduction ratio through many experiments.
The consistency of the hypothesis is much improved when 2D geometric effects are included.

Turbulence kinetic energy is difficult to measure near L-H transition due to quenching. This means avoidable error: not dimensionless as above, but the following integral avoids division by $\tilde{K}$.

$$\tilde{K}_{\min}^{L} = \int_{t_0}^{t_d} dt \left[ P + \partial_r \tilde{T} - \gamma_{\text{eff}} \tilde{K} \right]$$

- turbulence not homogeneous poloidally, while the zonal flow is;
- the radial region of transfer $\sim 1/5$ the correlation length of turbulence

Estimate with above 2D corrections (poloidal variation assumed $\cos \theta$)
L-H transition summary

- local GPI measurements confirm that the initial turbulence reduction at the L-H transition happens as a lossless power conversion to Zonal Flow.
- the observed total converted energy is more than enough to cause the entire amount of observed initial reduction of turbulence.
- the Reynolds drive is sufficient to explain the local growth of the low frequency fluid flow \textit{prior to} the formation of large diamagnetic flows.
- the recorded dynamics show a remarkable level of similarity to predator-prey type models of L-H transition, i.e. a lossless energy conversion mechanism with the turbulence driven ExB flow as a key player in triggering the transition.
Parametric dependence of turbulence nonlinearity
Approaching the L-H threshold

- Power threshold of the L-H transition is a central question
- Threshold condition with edge flows is confirmed
- The plasma leading up to the transition itself is in L-mode:
  - relatively cold edge
  - strong turbulence
- To predict threshold, scaling trends of nonlinearities are needed

Seed $E_r$ shear naturally exists even without external heating; but it evolves:
- increases with heating power
- nonlinearity is expected to follow this
- scaling against other parameters?
Nonlinear interactions measured in steady state

L-H transition is inherently non-stationary; L- and I-modes are steady state. Bispectral techniques can estimate nonlinear interactions:

\[ T_v(f_1, f) = -\Re \langle \tilde{v}_f^\theta \tilde{v}_r^{f-f_1} \partial_r \tilde{v}_f^{f_1} \rangle \]

ie. the Fourier decomposition of \( P \) – known as three-wave kinetic energy transfer function. The effective nonlinear GAM drive rate:

\[ \gamma_{NL} = \frac{\sum_{f_1} T_v(f_1, f)}{|v_\perp|^2(f)} \]

Once the dominant GAM drive is found in steady segment, some time resolution is recovered by constructing a quantity like \( P \), restricted to the GAM frequency in \( f \).
Structure of L-mode \( T_v(f_1, f) \)

Convergence of bicoherence

- Good sign of convergence
- Reasonable convergence by ~250 realizations
- Coupling \( (b^2) \) shows before drive \( (T_u) \) is significant or
- before I-mode (typical GAM regime in C-Mod) see below
Energy transfer rates increase with ICRF heating

The normalized amount of nonlinear coupling, i.e., the transfer into ZF increases monotonically with $P_{RF}$ at each value of plasma current in L-modes before reaching I- or H-mode.

![Graph showing energy transfer rate to ZF as a function of ICRH (MW)]

Data are restricted to experiments with similar average electron densities.
Sample spectra from ICRF L-mode w/ seed of GAM

- Transfer rates defined as sum of transfer function below some frequency for ZF, or
- over bandwidth of GAM
- At some value of ICRF heating power bicoherence spectra show clear peaks at GAM frequency without any significant transfer into GAM – dependence on \( I_p, n_e, B_t \) not understood so far
Access to I-mode and the L-I transition
Access to I-mode seems to be related to GAM

Drive term compared to collisional damping rate from neoclassical theory:

\[
\gamma = \frac{4}{7} \left( \frac{\nu_{\parallel}}{q} \right)
\]

\[
\gamma_{NL} = \frac{\sum f_1 T v (f_1^{GAM}, f)}{|v_\perp|^2(f)}
\]

At L-I transition, drive overcomes damping, due both to decreasing collisionality and enhanced drive.

At the I-H transition the drive becomes substantially smaller, and GAM disappears.
L-I transition on a fine time scale dissimilar to L-H

- no significant Dα drop (mass transport L-mode-like)
- beginning of heat pulse
- total turbulence power not affected
- onset of GAM (due to $T_e$ rise, $ν_{ii}$ drop)

\[- \frac{\langle ν_0 \partial_r (\tilde{ν}_r \tilde{ν}_θ) \rangle}{\langle (ν_0)^2 \rangle} \text{ P-like term}\]
Both GAM and edge coherent mode are crucial for I-mode

Recent experiments designed to study marginal I-modes resolve GAM-WCM degeneracy in I-mode onset:

C-mod:  
- GAM always present in I-mode
- WCM present
- coherent mode close to I-mode (seed of WCM without GAM)
- no GAM in L-mode or H-mode

DIII-D:  
- GAM are present in L-mode
- no high frequency coherent fluctuation
- I-mode accessibility range severely limited
Open Questions

**L-H transition**
- turbulence quenching occurs on a global scale, thus the behavior of the observed quantities on *the whole flux surface* matters; we need
  - more data from larger regions
  - simulations
- Connection to I-mode and H-mode-unfavorable grad$B$ drift directions in general.

**I-mode**
- origin of WCM - expect coherent mode!
- origin of the separation of transport channels:
  - high freq driving mass transport?
  - reduction of low freq component reducing heat transport?
- Why are low freq density fluctuations reduced?
- At what parameter ranges can difference btw WCM and GAM activity be further resolved?
- Is the ZF still the trigger in the I-H transition?
  - need correct ZF measurement in I-mode, which:
    - needs CXRS–GPI comparison,
    - precise alignment is an issue.