Full Wave Simulations of Fast Wave Mode Conversion and Lower Hybrid Wave Propagation

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November 4, 2003

Presented at 45th APS-DPP in Albuquerque
Fast, High Resolution Simulations are needed.

- Wave Physics
  - Modeling of Mode Converted Ion Bernstein Waves (IBW), Ion Cyclotron Waves (ICW), and Lower Hybrid (LH) Waves
  - Coupling to Antenna, Fokker-Planck Codes, Transport

- Experimental Modeling
  - Accurate Representation of Experimental Equilibria
  - Sufficient Resolution for Model Convergence, Advanced Scenarios

⇝ Algorithm Enhancements to TORIC
  - Parallelization of solution and post-processing (power and current deposition) - extends resolution and speed.
  - EFIT Equilibria - using the NTCC module, I2MEX.
Coordinates are aligned to Flux Geometry for Efficiency

**EFIT for Alcator C-Mod.**

Showing contours of constant flux, \((\psi)\), and poloidal angle, \((\theta)\).

**TORIC decomposition** is spectral in \(\theta\) and toroidal angle \((\phi)\) and finite elements in the flux dimension:

\[
E(x) = \sum_{m} E_m(\psi) \exp(im\theta + in\phi)
\]

with toroidal axisymmetry assumed.
TORIC is a Finite Larmor Radius Full Wave Code

- **TORIC** uses an FLR model for the plasma current response, \( J^P \).
- The antenna is modeled as a current sheet, \( J^A \), of a given poloidal distribution, radial location, and toroidal mode.
- It solves Maxwell’s equations for a fixed frequency with a linear plasma response in a mixed spectral-finite element basis.

\[
\nabla \times \nabla \times E = \frac{\omega^2}{c^2} \left\{ E + \frac{4\pi i}{\omega} (J^P + J^A) \right\} \quad \leftrightarrow \quad E(x) = \sum_m E_m(\psi) \exp(i m \theta + i n \phi)
\]

- **TORIC** uses the Swanson-Colestock-Kashuba approximation for the plasma response to the RF. The plasma current, \( J^P = \int \sigma(x,x') \cdot E(x) \), is in general an integral response. In **TORIC**, contributions through second harmonic are retained in the conductivity, \( \sigma \).
The TORIC Dielectric Model

- The TORIC dielectric models includes IBW, ICW, FW, and LH dispersion.

- Resolution needed depends on the specific wave scenario being modeled.

  - MC - presence of IBW implies $k_\perp \rho_i \approx 1$ and if $k_\perp \sim \frac{m}{r}$, then $M_{\text{max}} \geq \frac{1}{\rho^*} \approx 255$, for typical device parameters. ($\rho^* \equiv \rho_i/L$)

  - LHRF - ($\Omega_{ce}^2 \gg \omega^2 \gg \Omega_{ci}^2$), dispersion yields:

    $\frac{\omega_{pe}}{\omega} k_\parallel \sim k_\perp \sim \frac{m}{r} \Rightarrow M_{\text{max}} \sim 1000$

- For IBW, the imaginary part of the FLR $\sigma$, is modified with damping calculated from the full plasma conductivity with all orders of $k_\perp \rho_i$ and $\Omega_i$ retained.
TORIC’s Large Matrix can be inverted efficiently

\[ \mathbf{A} \cdot \mathbf{E} = \mathbf{J}_A \] where \( \mathbf{A} = \begin{pmatrix} \mathbf{D}_1 & \mathbf{U}_1 & 0 & 0 & 0 \\ \mathbf{L}_2 & \mathbf{D}_2 & \mathbf{U}_2 & 0 & 0 \\ 0 & \ddots & \ddots & \ddots & 0 \\ 0 & 0 & \mathbf{L}_{Nm-1} & \mathbf{D}_{Nm-1} & \mathbf{U}_{Nm-1} \\ 0 & 0 & 0 & \mathbf{L}_{Nm} & \mathbf{D}_{Nm} \end{pmatrix} \]

- Discretizing the BVP produces a matrix equation. The blocks, \( \mathbf{L}, \mathbf{D}, \) and \( \mathbf{U} \) are each dense matrices of size \( O(6Nm)^2 \).
- The individual \( 3N_r \) blocks are distributed across the processors and inverted using (SCA)LaPack to do an \( \mathbf{LU} \) decomposition.
- Processor memory limitations on simulation sizes are removed by using an out-of-core technique in which block inverses are stored on local disks.
Parallel Performance allows the Exploration of New Physics Regimes

- Non-rigorous comparison of serial and parallel run times with Marshall (the PSFC-MIT theory Beowulf cluster).

<table>
<thead>
<tr>
<th>$N_m$</th>
<th>Time(hours)</th>
<th>Machine</th>
</tr>
</thead>
<tbody>
<tr>
<td>161</td>
<td>10</td>
<td>Killeen 1 pc - vector SV1 CRAY</td>
</tr>
<tr>
<td>255</td>
<td>9.5</td>
<td>Marshall 1 pc - 2 GB Athlon 1.2 GHz</td>
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<tr>
<td>511</td>
<td>won’t fit</td>
<td>Marshall 1 pc</td>
</tr>
<tr>
<td>127</td>
<td>0.11</td>
<td>Marshall 32 pc (16 nodes)</td>
</tr>
<tr>
<td>255</td>
<td>0.78</td>
<td>Marshall 32 pc</td>
</tr>
<tr>
<td>511</td>
<td>3.6</td>
<td>Marshall 32 pc</td>
</tr>
<tr>
<td>1023</td>
<td>25.5</td>
<td>Marshall 48 pc</td>
</tr>
</tbody>
</table>
15 Poloidal Modes are insufficient to resolve MC Layer

- \( N_m = 15 \) for \( \text{D}(47\%) - \text{He}^3(24\%) - \text{H}(5\%) \) mode conversion case.

- Proper radial localization in a vertical layer IS NOT captured. Magnification of the layer shows a tendency to follow flux surfaces.

- Spurious minority damping \( \rightarrow \) power balance of 85% ions and 15% electrons. But, 1D fullwave and experiment predict mostly electron damping.
Power Spectrum measures Convergence at Higher Resolution

- $N_m = 127$ mode conversion case still has $\sim 10\%$ of peak amplitude in edge spectrum.
- Mode Conversion Region occurs at $\rho \approx 0.5$
- This verifies the convergence of the spectral expansion in the MC regime.

- $N_m = 511$ spectrum residual is $\approx 0.1\%$ even on outer flux surfaces.
At Higher Resolution, the Mode Conversion Layer is resolved

- At $N_m = 511$ the MC layer is well localized.
- Spurious damping is eliminated and about 88% of power is in the electron channel.
Dependence on $^3$He in C-Mod is improved.

- The higher resolution runs capture the trend of power deposition with increasing $^3$He concentration.
- At higher $^3$He concentration single pass absorption decreases.
- The overestimation may be due to power absorbed by the edge shear Alfvén resonance - a mode too small to be captured by the simulation.
Mode Conversion to ICW and IBW

- Alcator C-Mod discharge with mixture of D-$^3$He-H in (21%-23%-33%) of $n_e$ proportion.
- This scenario has long propagation of the ICW above and below the midplane back toward the low field side.
- The ICW up/down asymmetry due to shifting of $k_\parallel$ is clear (Perkins 1977.)
- Also, note the resolved short wavelength IBW on the midplane.
The IBW Propagation is well Resolved

- The IBW damping model allows the Ion Bernstein Waves to propagate stably.
- Propagation length is in agreement with 1-D models (AORSA-1D and Mets95)
- We can see the planar nature of the propagation along the midplane.
A Comparison with the AORSA Code

- The All ORders Spectral Algorithm Code has been developed at ORNL.

- It retains all orders in $k_\perp \rho_i$ in the plasma response,
  \[ J^P = \int \sigma(x,x') \cdot E(x). \]

- The geometry is cylindrical-toroidal, removing the coordinate center from the computational domain.

  \[ E(x,y,\phi) = \sum_{n,m,l} E_{n,m,l} \exp(i(k_n x + k_m y + l\phi)) \]

- The fully spectral approach facilitates implementing the all orders conductivity, but results in a fully dense matrix - and a much larger numerical problem.
Features of the MC Region are very similar in the two Codes

- **AORSA** at \(230N_x \times 230N_y\)
- **TORIC** at \(240N_\psi \times 255N_\theta\)

- Both codes are using the same equilibrium from an Alcator C-Mod discharge with mixture of D-\(^3\)He-H in (21%-23%-33%) of \(n_e\) proportion. Note that the location of the three waves and the MC layer are all very similar.
Global power balances differ:

- **TORIC**- 77% P(e)
  22% P(H)
- **AORSA**- 51% P(e)
  47% P(H)

- The **electron power depositions** are qualitatively similar.
- The **ion power depositions** have larger differences.
Differences in Power Deposition have two possible Causes

- **AORSA-1D** at twice the horizontal resolution \((N_x = 500)\), has a power balance nearly the same as TORIC in 2D.

- The validation of TORIC’s IBW damping model still remains - though the IBW accounts for only a few % of total electron damping in this case.

- It should be noted for parameters with smaller \(k_\parallel \rho_i\), such as MC in DIII-D, AORSA has demonstrated convergence in 2D.
TORIC has been implemented in the LHRF Regime and initial results are encouraging

- TORIC has been run in the ICRF regime with up to \( N_m = 1023 \times N_\psi = 480 \). This is the resolution required to resolve the slow LH wave.

- The conductivity operator has been rewritten to be valid for \( \Omega_{ci}^2 \ll \omega^2 \ll \Omega_{ce}^2 \).
  - Unmagnetized ions - analytically equivalent to \( (k_\perp \rho_i)^2 \to \infty \) limit.
  - Strongly magnetized electrons - \( (k_\perp \rho_e)^2 \ll 1 \)

- Presently the code couples to the FW polarization in the LHRF regime \( (E_\parallel = 0 \text{ at antenna}) \)

- Work is underway to modify the boundary conditions to couple to the slow wave \( (E_\parallel \neq 0 \text{ at antenna}) \)
• **TORIC** has resolved the FW polarization at LH frequencies, \( \omega = 4.6 \) GHz, \( n_\phi = 240 \).

• The power deposition profile is that of the fast wave. Damping strength follows the plasma beta and shifts as the central temperature is changed.

• FW coupling is also consistent with the dispersion relation.
Conclusions

1. Well converged simulations of mode-conversion scenarios are now routine, and provide a fast, flexible tool for studying current and flow profile control through mode-conversion.

2. We can model advanced scenarios such as ITER or burning plasmas with the new resolution capabilities. Coupling to other codes such as RANT3D to study antenna coupling, or function as a module in the TRANSP code suite for studying MC effects on transport.

3. TORIC has sufficient resolution and the proper dielectric for lower-hybrid full wave simulations. Questions such as the role of wave focusing and diffraction in LH spectral broadening may yield new insights to the full wave model.