Measurement and Modeling of Alfvén Cascades on Alcator C-Mod

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Introduction

Alfvén cascades, also known as Reverse Shear Alfvén Eigenmodes (RSAE), are a type of MHD instability driven by a gradient in the fast ion pressure. The frequency of these modes is sensitive to small changes in $q_{\text{min}}$, the minimum of the safety factor. A slightly reversed shear profile can aid the formation of the cascade modes.

Current ramping experiments on Alcator C-Mod with early RF heating have produced a flat or slightly hollow q-profile before steady state current is reached. The ideal MHD code NOVA with its kinetic extension NOVA-K is being used to calculate the mode frequency and structure. With the additional information of the fast ion distribution, the stability of these modes can be calculated.

With information on the time evolution of the Alfvén cascade frequency, the toroidal mode number $n$, and the plasma parameters, the results from NOVA can be fit to experimental data leaving as the only variable the q-profile. In this way Alfvén cascades can be used as a tool for modeling the time evolution of $q_{\text{min}}$.

Diagnostics

The Phase Contrast Imaging (PCI) diagnostic measures line integrated electron density perturbations.

PCI has 32 channels along the major radius covering about 12 cm through the core. Digitizers are capable of 5 MHz frequency resolution. The current PCI system on C-Mod can measure $k_R$'s in the range $0.5 \text{ cm}^{-1}$ to $8 \text{ cm}^{-1}$.

Measured signal intensity is proportional to the magnitude of the density fluctuations.

Frequency Characteristics of Alfvén Cascades

Sharapov et al. described in ref [3] the measurement of Alfvén cascades on JET discharges. They developed the following empirical formula to describe the evolution of the frequency,

$$f(t) = \frac{1}{2\pi} \left( \frac{m}{q_{\text{min}}(t)} - n \right) \frac{V_i}{R_0} + f_0$$

where $f_0$ is a constant offset, dependent on $\beta$.[9] The cascade modes have a single dominant poloidal harmonic.

Closely related to the cascades is the Toroidicity-induced Alfvén Eigenmode (TAE). The frequency of this mode is given by [10]

$$f_{\text{TAE}} = \frac{n}{2(2m \pm 1)} \frac{V_i}{2\pi R_0}$$

A cascade will evolve into a TAE as it approaches the TAE frequency. A TAE is distinguished by two or more dominant poloidal components, and a rather steady frequency.

The 32 channel PCI system can resolve the spatial structure of modes in high time resolution.

The cascade/TAE transition is clearly marked by a movement of peak intensity from outboard side to inboard side, magnetic axis is ~ 67 cm.

The spatial structure broadens during the TAE phase (notice also the bottom plot in the group of three on the next page).

A great amount of information about the evolution of the q profile can be found in these modes. This may allow greater constraints to be placed on the q profile, giving information on the shape near the $q_{\text{min}}$ surface.

The TAE’s appear with large amplitude on magnetics data. This is consistent with the picture that TAE's have a more global structure.
Measurement of Second Harmonic Cascade

In many shots, second harmonics were observed. The magnitude of these modes is at least an order of magnitude below the first harmonic.

The spatial structure of the second harmonic is seen to be similar, though not identical to that of the first harmonic.

The existence of the second harmonic is supported by magnetics data.

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Due to the finite width of the cascades there is an uncertainty in the ratios of about ±0.5.

Avoiding the ends of the cascades where there may be a non-linear response in frequency to changes in q, we take the ratio of the second to the third cascade as 1.5. The ratio of the first to the second cascade is very nearly 2.0. This gives as a likely set of n numbers \(\{1,2,3,4\}\), with the qualified assumption that the lowest n number is 1.

Modeling Cascade Activity with NOVA

NOVA uses a full toroidal model and plasma shape to calculate the eigenmode structures and frequencies.

Inputs for NOVA are experimental the density profile, temperature profile, plasma shape parameters, plasma composition, and an estimated q profile. Though the q profile is not known exactly, certain features (q_{min} its radial position, and its time evolution) can be inferred from matching NOVA cascade solutions to modes measured by PCI and magnetics.

Most effort on modeling cascades has focused on shot 1040406018, where a very clear grand cascade allows determination of the n numbers from the slopes.

Below are the results of a first attempt to model the q_{min} evolution. The NOVA calculated mode frequencies are shown in white, using dq_{min}/dt = 0.011 ms\(^{-1}\).

Better matching of the frequencies may be achieved by varying the input parameters. Inherent uncertainty in the q profile, and experimental error in the density and temperature profiles allow for significant frequency and mode structure variations.

Shown below are NOVA calculated eigenmodes for the n=2 and n=3 cascades at a q_{min} of 1.85. Sqrt(T_{poloidal}) = 0.15 \quad n/a = 0.12.

Conclusion

PCI is a unique diagnostic in its ability to measure high resolution, core-localized density fluctuations.

PCI’s large range of frequency sensitivity has allowed for the observation of second harmonic modes.

The time development and spatial structure of core modes, like Alfvén cascades, can be measured with PCI.

On C-Mod, the n numbers are difficult to measure, but can be inferred during grand cascades from the slopes.

NOVA is being used to model the Alfvén cascade eigenmode development, with the goal of modeling the evolution of the q profile.

Measurement of the spatial structure of these modes may be useful in constraining the shape of the q profile around q_{min}.

Future Work

Understanding where the modes are located may prove extremely valuable in determining not only q_{min} but also the q profile around q_{min}. To do this we need to determine how to deconvolute the line-integrated signal. To do this we first need to answer, what is the k-space composition of the modes?

Mode stability calculation from NOVA-K, including the fast ion distribution, will further support, or refute, the eigenmode solutions from NOVA-K as viable modes.

Current efforts by the magnetics diagnostic team may prove fruitful in determining the n numbers for many more modes, allowing more complete modeling of the q profile evolution.

References

4. M. Porkolab et al., EPS meeting, Berchtesgaden, Germany (1997).

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