Experimental studies of RSAEs on Alcator C-Mod

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Hollow current density profiles form as the Ohmic current diffuses toward the core

- Finite resistivity means that the current diffuses toward the core on a resistive time scale, in Alcator C-Mod this is
  \[ \tau_\eta \sim \frac{\mu_0 L^2}{\eta} \sim a^2 T_e^{3/2} \sim 200 \text{ ms (startup)} \]
  \[ \sim 30 \text{ ms (sawteeth)} \]

- ITER may achieve temperatures of 20+ keV during the current ramp
  \[ \tau_\eta \sim a^2 T_e^{3/2} \sim 300 \text{ sec (startup)} \]

- Alfvén eigenmodes will likely be driven by beams, ICRH and \( \alpha \)-particles
Reversed shear Alfvén eigenmodes (RSAEs) observed during the current ramp and sawteeth

- RSAEs observed during the current ramp
  - MHD spectroscopy
  - No sawteeth
  - Good regime to study energetic particle transport?

- RSAEs observed during sawteeth
  - Both L and H mode
  - Significant differences in RSAE excitation
  - $T_e > 3$ keV
  - $n_{e0} < 1.5 \times 10^{20}$ m$^{-3}$
The reversed shear Alfvén eigenmodes exhibit strong dependence on $q_{\text{min}}$

\[
\omega^2_{\text{RSAE}} \approx \frac{V_A^2}{R_0^2} \left( \frac{m}{q_{\text{min}}(t)} - n \right)^2 + \frac{2T_e}{M_i R_0^2} \left( 1 + \frac{7}{4} \frac{T_i}{T_e} \right) \\
- \frac{2}{M_i R_0^2} r \frac{d}{dr} T_e \left( 1 + \frac{T_i}{T_e} \right) - \frac{\omega_{\text{CH}} \omega_{\text{RSAE}}}{m_{\text{poloidal}}} r \frac{d}{dr} \left( n_H \right)
\]

- $0\text{-}500 \text{ kHz}$
- $\sim 200 \text{ kHz}$
- $\sim 100 \text{ kHz}$
- $< 100 \text{ kHz}$

Breizman et al., PoP 12, 112506 (2005) [kinetic effects]
Gorelenkov et al., PPCF 48, 1255 (2006) [MHD derivation]
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\[- \frac{2}{M_i R_0^2} r \frac{dT_e}{dr} \left( 1 + \frac{T_i}{T_e} \right) - \frac{\omega_{\text{CH}} \omega_{\text{RSAE}}}{m_{\text{poloidal}}} \frac{r}{n_e} \frac{d}{dr} \langle n_H \rangle \]

- 0-500 kHz
- ~200 kHz
- ~100 kHz
- < 100 kHz ??

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Outline

- Phase contrast imaging and Mirnov coils
- RSAE minimum frequency scaling
- RSAE tunneling and edge penetration
- Observations during sawteeth
Phase contrast imaging and Mirnov coils are the primary tools for Alfvén wave studies in C-Mod.

**Phase Contrast Imaging (PCI)**
- PCI measures path-integrated electron density fluctuations

\[ I_{PCI} \sim r_e \lambda_{laser} \int \tilde{n}_e \, dl \]

- 32 channel detector measures spatial structure between 0.6 m < R < 0.8 m
- Frequency Range
  \(2\text{kHz} < f < 5\text{ MHz}\)

**Mirnov Coils**
- Coils at 6 toroidal locations are used to identify low-n modes

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NOVA\textsuperscript{1} results are compared to experiment with a synthetic PCI analysis\textsuperscript{2}


\textsuperscript{2}Synthetic PCI developed in collaboration with Gerrit Kramer.
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Minimum Frequency Scaling

Minimum frequency expected to scale with $T_e$

$$f_{\text{min}}[\text{kHz}] \approx 100 T_e^{1/2} \sqrt{0.75 \gamma + 0.15 \frac{a}{L_T}}$$

This is the NOVA scaling.

$$f_{\text{min}}[\text{kHz}] \approx 70 T_e^{1/2} \sqrt{\left(1 + \frac{7 T_i}{4 T_e}\right) + 0.3 \left(1 + \frac{T_i}{T_e}\right) \frac{a}{L_T}}$$

This is the Breizman et al. scaling.

Agree within 10% when $T_i/T_e = 0.8$
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MHD equation of state

\[
\frac{d}{dt} \left( \frac{p}{\rho^\gamma} \right) = 0
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PCI measurements support
$1.25 \leq \gamma \leq 1.55$

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For shear Alfvén waves\(^1\),
(no geometric effects)

\[ \nu_1 \approx \frac{B_1}{\sqrt{\mu_0 \rho}} \]

\[ U = \frac{1}{2 \mu_0} B_1^2 + \frac{1}{2} \rho v_1^2 \]

\[ = \frac{1}{\mu_0} B_1^2 \]

\[ P = \frac{1}{2 \mu_0} B_1^2 I - \frac{1}{2 \mu_0} B_1 B_1 + \rho v_1 v_1 \]

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\[ P = (\gamma - 1)U \quad \rightarrow \quad \gamma = \frac{3}{2} \]

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Kinetic electrons:

$$\frac{v_{th,e}}{v_A} \sim 2$$

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Q: How does tunneling affect the edge amplitude?
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A: #1 Compare PCI and magnetics
   #2 Simulate the effect with NOVA
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Future experiments may be able to test this effect over a wider range of conditions.

Alfvén continuum should vary strongly with:
- plasma current
- edge density
- plasma shaping?
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An additional barrier is presented to the eigenmodes in the case with larger edge $q$. 
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Alfvén continuum should vary strongly with:
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- plasma shaping?

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Edge amplitude decreases.
RSAEs excited during sawteeth show both up-chirping and down-chirping patterns.
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q=1 RSAEs are observed with PCI and magnetics

- RSAEs during sawteeth have been observed with PCI and Mirnov Coils.
- PCI spectra are used to infer mode numbers from
  \[ f_n^2 = f_0^2 + f_A^2 \left( \frac{m}{q_{\text{min}}} - n \right)^2 \]
- Mirnov coils directly measure the toroidal phase \( m = n \) near \( q = 1 \).

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Down-chirping RSAEs constrain the post-crash q profile

- Faint down chirping RSAEs are frequently observed during sawteeth
- Strong down-chirping RSAEs followed a large sawtooth crash
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- Faint down chirping RSAEs are frequently observed during sawteeth
- Strong down-chirping RSAEs followed a large sawtooth crash
- Mode numbers and spatial structure can be clearly identified

- What q profile following the sawtooth crash can result in down-chirping RSAEs?
Down-chirping RSAEs imply a local maximum in the q profile following the sawtooth crash

- NOVA-K finds regular RSAE solutions with $q < 1$ (pre-reconnection)
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- NOVA-K finds regular RSAE solutions with $q < 1$ (pre-reconnection)

- No solutions exist for a conventional reversed shear q profile with $q_{\text{min}} > 1$

- RSAE solutions do exist when the q profile has a local maximum
Down-chirping RSAEs imply a local maximum in the q profile following the sawtooth crash.
Observation of RSAEs constrains the evolution of the q profile during sawteeth

- Post-reconnection q profile has a local maximum
  - down-chirping RSAEs

- Pre-reconnection q profile has reversed shear in the region r/a < 0.2
  - up-chirping RSAEs

- A model of the sawtooth cycle requires a relaxation process and reconnection process that closes the cycle.
• RSAEs observed during the current ramp phase
  – Relatively quiescent window for the study of the interplay between RSAEs and energetic ions
  – The coupling of core modes to the magnetics needs more work
    • Experiments underway, more modeling needed
  – Minimum frequency bounds the adiabatic index to $1.25 \leq \gamma \leq 1.55$
    • Are energetic ion contributions important?
    • How to model the plasma compressibility in the limit $k_{||} \rightarrow 0$?
    • Can an effective adiabatic index be derived from simulation?

• RSAEs frequently observed during sawteeth with $n_{e0} < 1.5 \times 10^{20} \text{ m}^{-3}$
  – Experiments at C-Mod conducted in ITER relevant conditions
  – RSAEs offer the possibility of core MHD spectroscopy during sawteeth
  – Down-chirping RSAEs suggest a local maximum
  – Kadomtsev model could possibly explain the observed RSAEs
Evidence of non-linear RSAE harmonics

Harmonic frequency range

Fundamental frequency range
$2^{nd}$ order perturbations represent mode coupling of like and unlike toroidal mode numbers.
Phase contrast imaging transforms phase variations to intensity variations

Without phase plate

\[ I_{PCI} \sim E_0^2 \left| 1 + i\Delta \right|^2 \sim E_0^2 (1 + \Delta^2) \]

With phase plate

\[ I_{PCI} \sim E_0^2 \left| i + i\Delta \right|^2 \sim E_0^2 (1 + 2\Delta) \]

\[ \Delta(x) = \frac{\omega_0}{c} \int \tilde{N}(x, z) \, dz \]

\[ = \left( \frac{e^2}{4\pi\varepsilon_0 m_e c^2} \right) \lambda_0 \int \tilde{n}_e(x, z) \, dz \]

\[ \Delta(x) = r_e \lambda_0 \int \tilde{n}_e(x, z) \, dz \]
The Kadomtsev model can produce a local maximum in $q$

- Start with Kadomtsev Model\(^1\) for reconnection
- Surfaces of equal helical magnetic flux reconnect
- Toroidal magnetic flux is conserved

\[
q(r) = \frac{1}{1 + \frac{1}{2\pi r} \frac{d}{dr} \Psi^*}
\]

safety factor

\[
\Psi^* = \int_{0}^{r} \left( \frac{1}{q(r')} - 1 \right) 2\pi r' dr'
\]

helical magnetic flux

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- Start with Kadomtsev Model\(^\text{1}\) for reconnection
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A local maximum in $q$ can be produced when the reconnection starts from a reversed shear $q$ profile

\[ q(r) = \frac{1}{1 + \frac{1}{2\pi r} \frac{d}{dr} \Psi^*} \]

\[ \Psi^* = \int_0^r \left( \frac{1}{q(r')} - 1 \right) 2\pi r' dr' \]