Faraday Effect Measurement of Internal Magnetic Field and Fluctuations in C-MOD

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Motivation

• Magnetically confined fusion devices require detailed time-resolved measurement of \( J(r) \) and \( B(r) \):

\[
\vec{J} \times \vec{B} = \nabla P
\]

• Measurements of \( \delta B \) and \( \delta J \) associated with instabilities (MHD, fast-particle modes, turbulence, disruptions) are critical to understanding transport

• **Goal for C-Mod**: pursue polarimeter development for both magnetic equilibrium and fluctuation measurements

• **ITER q(r) diagnostic**
  - C-Mod has ITER-like (\( B, n_e \)), polarimetry geometry (double-pass) and wavelength (118 \( \mu \)m)
Outline

(1) Measurement principle
   - Faraday effect ($\vec{B} \parallel \vec{k}$)
   - Cotton-Mouton effect ($\vec{B} \perp \vec{k}$)

(2) Key factors for Faraday rotation measurements on high field tokamak
   - Polarization distortion by optical components
   - Phase noise (vibrations, stray dB/dt, feedback, electronic)
   - Misalignment to toroidal field

(3) Faraday effect measurements
   - Equilibrium
   - Fluctuations

(4) Future Plans
Faraday Rotation

Polarization rotation due to parallel magnetic field (\( \vec{B} \parallel \vec{k} \))

\[ \psi_F = 2.62 \times 10^{-13} \lambda^2 \int n_e B_z \, dz \]

Faraday phase shifts are in the 0-30 degree range in C-Mod (118 \( \mu m \))
Faraday Rotation

Polarization rotation due to parallel magnetic field \( \vec{B} \parallel \vec{k} \)

Linearly polarized light can be thought of as the sum of R- and L-waves

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Faraday Rotation (Effect)

R- and L- waves are offset in frequency

\[ \psi_F = 2.62 \times 10^{-13} \lambda^2 \int n_e B_z \, dz \quad \Rightarrow \quad \frac{\omega}{c} \int \frac{(n_R - n_L)}{2} \, dz \]

phase measurement
Cotton-Mouton Effect

Polarization elliptized due to perpendicular magnetic field

\[ \psi_{CM} = 2.45 \times 10^{-11} \lambda^3 \int n_e B^2 \, dz \]
\[ \Rightarrow \frac{\omega}{c} \int (n_X - n_O) \, dz \]

1) C-M phase shifts up to 20° possible in C-Mod (118 µm)
2) For tokamaks, \( B \) has components parallel and perpendicular to the direction of the electromagnetic probe wave
Faraday Effect Detection Technique

\[ \lambda/2 \text{ Plate} \]

Reference Mixer

\[ \Omega_2 \]

\[ \Omega_1 \]

\[ \Omega_1 \]

\[ \Omega_2 \]

\[ \lambda/4 \text{ Plate} \]

\[ \text{Plasma} \]

\[ B_p \]

\[ B_T \]

Retro-reflector mounted in inner wall

FIR LASERS - (2.54 THz)
Frequency offset \( \sim 4 \text{ MHz} \)

**Phase Measurement:** effect of perpendicular \( B_T \) on \( R \)- and \( L \)-waves is identical and cancels, **no error due to Cotton-Mouton effect**

FIR Optical Layout

- Two CW far infrared lasers (Coherent Inc.: $\lambda = 117.73 \, \mu m$, $\sim 150 \, mW/cavity$). Frequency offset $4 \, MHz$ ($<1 \, \mu s$ time response)

- $\sim 14 \, m$ pathlength

- Not shown:
  - Air-tight enclosures for humidity control
  - 1.2 cm thick magnetic shield for laser stability

C-Mod:

- $B_T = 3 - 8T$
- $I_p = 0.4 - 2.1MA$
- $n_e = 0.3 - 5.0 \times 10^{20} \, m^{-3}$
- $R = 0.68 \, m$
- $a = 0.22 \, m$
Main components on upper table:

- ¼ wave plate
- TPX lens -> focus beam on corner cube reflector
- low-noise planar diode mixers
- impact parameter: $x_1 = 10$, $x_2 = 16$, and $x_3 = 20$ cm
Mesh Beam Splitters May Alter Beam Polarization

For an incident angle of 45°, linearly polarized light may become elliptically polarized \textit{AFTER} reflection / transmission through beam splitter.

Effect depends on:

1) Orientation of wire grid with respect to incident beam polarization (drawn)

2) Wire grid density (not drawn)

\textit{Both errors minimized in optical layout}
Rotating $\lambda/2$ plate causes phase change.

- Measured phase difference is **linear** if polarization of R- and L- waves are not modified by optics (mesh).
Measured Phase Response

**Linear response:** beam is comprised of R- and L-circularly polarized waves

**Non-linear response:** phase change from optical components

Calibration curves can correct for systematic errors
**Measurement Phase Noise**

Acoustically vibrations in lasers: 0.25° rms (500 kHz bandwidth)

- Laser relocation addresses both noise sources
- Optical feedback to lasers below noise level
Toroidal Field Effects:

(1) Alignment and (2) Cotton-Mouton effect

1) Misalignment to toroidal field, $B_T$

$$\psi_F \propto \int n_e B_z \, dz = \int n_e B_P \cos \alpha \, dl + \int n_e B_T \sin \alpha \, dl$$

2) Cotton-Mouton effect

$$\psi_{CM} \propto \int n_e B_T^2 \, dl$$

- C-Mod toroidal magnetic field 4 – 8T

- Dedicated tests required to determine effect on measured phase shift
Negligible $B_T$ Effect on Faraday Measurement

$n_e$ and $I_p$ same: change only $B_T$

No change in measured phase

$\rightarrow$ No C-M effect $\rightarrow$ No $B_t$ effect

$B_t = 7.5 \text{ T}$

$B_t = 5.4 \text{ T}$

C-M effect cancels when using R-, L-wave approach
Equilibrium Faraday Effect

\[ \psi_F \propto n_e \rho B \cdot dl \]

Measurements can be used to constrain magnetic equilibrium reconstruction
Plasma Dynamics Captured during Lower Hybrid Current Drive

- Faraday signal evolves with LHCD
- Indication for $J(\rho)$ change
- Further analysis, modeling, and verification ongoing

\[ \psi_F \propto \int n_e \vec{B} \cdot d\vec{l} \]

- $x_1 = 10 \text{ cm}$
- $x_2 = 16 \text{ cm}$
- Lower hybrid power (kw)

\[ n_e l \times 10^{20} \text{ m}^{-2} \]
Cotton-Mouton Measurement

\[ \psi_{CM} = 2.45 \times 10^{-11} \lambda^3 \int n_e B_\perp^2 \, dz \implies \frac{\omega}{c} \int (n_X - n_O) \, dz \]

Fuchs and Hartfuss, PRL 81, 1626-1629 (1998)  
Akiyama, et al., RSI 77, 10F118 (2006)
Cotton-Mouton and Faraday effect measurements consistent with EFIT

1) Cotton-Mouton and Faraday effect measurements consistent with EFIT

2) C-M effect can be used to measure density as $B_T$ is known

$$\psi_{CM} \propto \int n_e B_T^2 dz$$

$$\Rightarrow \frac{\omega}{c} \int (n_X - n_O) dz$$

$$\psi_F \propto \int n_e B_z dz$$

$$\Rightarrow \frac{\omega}{2c} \int (n_R - n_L) dz$$
Faraday Fluctuation Measurement

\[ \psi \propto \int n_e \vec{B} \cdot d\vec{l} \]

\[ \psi = \psi_0 + \delta \psi \quad n = n_0 + \delta n \quad B = B_0 + \delta B \]

\[ \delta \psi = \int \delta n_e \vec{B} \cdot d\vec{l} + \int n_e \delta \vec{B} \cdot d\vec{l} \]

Term

Other fluctuation diagnostics aid in differentiating terms in

- PCI, reflectometer, scattering, \textit{interferometer (3rd laser)}
- external magnetic coils
Discrete and Broadband Fluctuations in Faraday Signal

Coherent feature at 80-100 kHz
~0.2^0 amplitude

Broadband feature at 400-800 kHz
~0.1^0 amplitude

C-Mod polarimeter has time and phase resolution required for fluctuation measurements
Energetic Particle driven mode during ICRF 200-400 kHz

- Low density shot with 3+ MW ICRF
- Spike in activity seen in $\delta n$ and $\delta B$ diagnostics
- Evidence for Alfvénic activity
Plans (Three Year Timescale)

- Locate lasers remotely from tokamak cell [temperature controlled]
  - Lower stray $B$ and acoustic noise (vibration)
  - Use dielectric waveguide to transport beams

- Increase number of chords from 3 to 10 (depending on access)
  - Horizontal view with one chord below midplane
  - Expand port for radial axis (more chords, $\delta B_r$)

- Add Density measurement capability
  - Exploit Cotton-Mouton effect
  - Add 3rd laser (order on HOLD)
  - magnetic and density fluctuations simultaneously resolved

- Leverage technique (fluctuation and equilibrium) from C-Mod to address long-pulse AT operations in EAST (preparation for ITER)
Summary

✓ Three chords operational in ‘12 campaign
  -> Measurements and EFIT reconstructions consistent
✓ Faraday measurement checked against Cotton-Mouton and $B_t$ effects
  -> No contamination found
  ▪ Fluctuations related to MHD, fast-particles instabilities and turbulence observed
  ▪ Future plans to improve laser stability, increase chords
  ▪ Results increase confidence in Faraday effect measurement for ITER
### Table 1: Combined Polarimeter-Interferometer Diagnostic

<table>
<thead>
<tr>
<th>Plasma Parameters</th>
<th>Multifield quantities</th>
<th>Physics Issues</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Interferometry</strong></td>
<td>Electromagnetic Torque$^2$</td>
<td>Equilibrium Dynamics</td>
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<tr>
<td>$n_o(r,t)$, $\tilde{n}(k,\omega)$</td>
<td>electrons: $\left\langle \tilde{j} \times \tilde{b} \right\rangle_{\parallel} / e n_e$</td>
<td>- Ohm’s Law$^7$</td>
</tr>
<tr>
<td><strong>Differential Interferometry</strong></td>
<td>ions: $\left\langle \tilde{j} \times \tilde{b} \right\rangle_{\parallel}$</td>
<td>- 3D effects$^8$</td>
</tr>
<tr>
<td>$\nabla n_o(r,t)$, $\nabla \tilde{n}(k,\omega)$</td>
<td>Charge Flux (Maxwell stress)$^3$</td>
<td>Momentum Transport</td>
</tr>
<tr>
<td><strong>Polarimetry - Faraday Effect</strong></td>
<td>$\Gamma_q = \Gamma_i - \Gamma_e = \left\langle \tilde{j}_{\parallel} \tilde{b}_r \right\rangle / eB$</td>
<td>- Maxwell stress</td>
</tr>
<tr>
<td>$J_\phi(r,t)$, $B_\theta(r,t)$</td>
<td>Particle Flux (divergence)$^4$</td>
<td>Electric Field Generation</td>
</tr>
<tr>
<td>$\tilde{b}<em>r(k,\omega)$, $\tilde{b}</em>\theta(k,\omega)$, $\tilde{j}_\phi(k,\omega)$</td>
<td>$\Gamma_e = \left\langle \tilde{j}_{\parallel} \tilde{b}<em>r \right\rangle / eB = \frac{V</em>{\parallel e}}{B} \left\langle \tilde{n} \tilde{b}_r \right\rangle$</td>
<td>- Zonal flows</td>
</tr>
<tr>
<td><strong>Collective Scattering</strong></td>
<td>Momentum Flux (Kinetic stress)$^5$</td>
<td>Particle Transport</td>
</tr>
<tr>
<td>$\tilde{n}(k,\omega)$</td>
<td>$\Gamma_{\text{ion}} = \frac{\left\langle \tilde{p}_{\parallel,\text{ion}} \tilde{b}<em>r \right\rangle}{B} \Rightarrow T</em>{\parallel,\text{i}} \left\langle \tilde{n} \tilde{b}_r \right\rangle$</td>
<td>Momentum Transport</td>
</tr>
<tr>
<td></td>
<td>Nonlinear 3-Wave Coupling$^6$</td>
<td>- Intrinsic flow</td>
</tr>
<tr>
<td></td>
<td>$\left\langle \tilde{n} e \tilde{v}_{e,r} \nabla \tilde{n}_e \right\rangle \Rightarrow \left\langle \tilde{n} e \tilde{b}_r \nabla \tilde{n}_e \right\rangle$</td>
<td>- Kinetic stress</td>
</tr>
<tr>
<td></td>
<td></td>
<td>driving/damping mechanisms for density fluctuations</td>
</tr>
</tbody>
</table>
End of Talk
Sawtooth Precursors in Faraday Signal

Sawtooth cycle seen $x = 16$ cm

$x = 10$ cm chord largely $\perp$ to $B_p$: measures

$\int n_e \delta B \, dl$
Impurity Driven Snake and Quasi-Coherent Mode (QCM) Observed

- Snake is long lived core helical perturbation
  \( \sim 20 \text{ kHz}, 0.5^0 \)

- QCM is a common component in ICRF heated plasmas
  \( \sim 70 \text{ kHz}, 0.4^0 \)
Chords Through or Near the Magnetic Axis Measure $\delta B_r$

Through the magnetic axis,

$$\int n_e B_P \cos \alpha dl = 0$$

The residual fluctuation measurement is largely

$$\int n_e \delta \vec{B} \cdot d\vec{l}$$

with $\delta \vec{B} \approx \delta B_r$

because the chord is along the magnetic axis

$$\int \delta n_e \vec{B} \cdot d\vec{l}$$

(term peaks off axis)

*Topic being considered as a future polarimeter upgrade*

**chord**
Deconstructing Faraday Phase Change

R + L wave electric field

Plasma

Phase difference between R- and L-waves can be measured parallel and perpendicular with respect to toroidal field direction

X direction (parallel to toroidal field) is typically selected, but component perpendicular to toroidal field (Y) can also be measured
- Toroidal component (X) ≈ perpendicular component (Y) at 5T and 8T - suggests no contamination by CM effect
- L – R wave Faraday probe technique is insensitive to CM effect
- C-MOD provides good test due to high field and density
Planar-Diode 2.8 THz Mixers

- Planar Schottky diode technology
- **No fragile whisker contacts**
- Operate interference frequency at 4MHz
- Bandpass sensitivity up to 1MHz, limited by filters
- 100-200 V/W
- **High-Sensitivity and low noise** allows access to fluctuation data

**Mixer Housing**
1.5” X 0.75”

**Horn**
0.27mm X 0.27mm
Key Optical Components

Custom mounts
- not easily tunable
- low vibration

Thin TPX lens focus beam onto retro reflectors
- 3/16” thick 4” OD
- 85% transmission
- allows for significantly shorter beam path
Angle of Mesh Grid Can Alter Beam Polarization

Proper mesh LPI and orientation W.R.T incident beam polarization required to properly split beam for multiple chords without altering polarization
Impurity Driven Snake phenomenon
Observed

• Snake is long lived helical perturbation in plasma core
• $\text{Mo}^{32+}$ presence allows for identification through SXR
• Snake apparent in chord #1 Faraday data as a $0.5^0$ oscillation in raw data