Measurement of lower hybrid waves using microwave scattering technique in Alcator C-Mod

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19th topical conference on radio frequency power in plasmas, June, 2011
Outline

- Motivation
- Development of 1D analytic model of scattering process in terms of non-linear coupling
- Hardware development to diagnose density fluctuation by LH waves
- Measurements of lower hybrid waves using reflectometry
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  2. LH power dependence in H-mode plasma
  3. Observation in high density L-mode plasma
- Summary and future work
Motivation: Direct measurement of lower hybrid waves near separatrix, using reflectometry

Lower Hybrid Current Drive (LHCD) has been proposed for non-inductive steady-state operation of tokamaks.

In Alcator C-Mod, LH waves (~1MW) are injected into plasma at 4.6GHz via an 64-waveguide grill.

Understanding the role of scrape-off layer (SOL) on the LHCD efficiency requires to directly measure LH wave fields.

We propose to use a existing reflectometer system in Alcator C-Mod.

We need to understand the scattering process in reflectometry in order to use wave-wave interactions as a diagnostic tool.
Development of 1D analysis model
Dispersion relations of probe waves and LH waves are dependent on plasma parameters, but mostly on density profiles.

- **Probe waves:** ordinary (O)-mode waves
- **LH waves:** electrostatic slow waves: backward waves
  - Dominant radial number
  - Parallel wave-number is conserved in the first order.

\[
\omega_{lh} \approx \omega_{pe} \frac{k_{par}}{k_{per}}
\]

In the case when probe beam interacts with LH waves leaving LH launcher [1]:

- 0: scattered waves
- 1: LH waves (4.6 GHz)
- 2: incident waves (60 GHz)
Dispersion relations allow only a single point to be resonant in an inhomogeneous plasma.

- Two scattering processes:
  - Frequency up-shifted
    \[ \vec{k}_0(x) = \vec{k}_1(x) + \vec{k}_2(x) \]
    \[ \omega_0 = \omega_1 + \omega_2 \]
  - Frequency down-shifted
    \[ \vec{k}_0(x) = \vec{k}_1(x) - \vec{k}_2(x) \]
    \[ \omega_0 = \omega_1 - \omega_2 \]
- Scattering occurs in front of cut-off layer.
We describe the scattering process in terms of non-linear mode coupling in 1D geometry.

- Each mode (two O-mode waves and LH wave) is a linear mode, but they become coupled to due to non-linearities.
- We assume three waves take the coupled form [2]

\[
\begin{pmatrix}
\vec{E} \\
\vec{H} \\
\vec{v}_i \\
\vec{v}_e
\end{pmatrix}
= a_0(t, \vec{r})
\begin{pmatrix}
\vec{E} \\
\vec{H} \\
\vec{v}_i \\
\vec{v}_e
\end{pmatrix}_0 + e^{i\Lambda_0} + CC + a_1(t, \vec{r})
\begin{pmatrix}
\vec{E} \\
\vec{H} \\
\vec{v}_i \\
\vec{v}_e
\end{pmatrix}_1 e^{i\Lambda_1} + CC + a_\gamma(t, \vec{r})
\begin{pmatrix}
\vec{E} \\
\vec{H} \\
\vec{v}_i \\
\vec{v}_e
\end{pmatrix}_\gamma e^{i\Lambda_\gamma} + CC
\]

- Time-averaged energy density
- Normalize mode vectors
- Power density

\[a_n\text{ : Slowly varying complex mode-amplitude in time and space}\]

\[\Lambda = \omega t - \vec{k} \cdot \vec{r}\]

\[a_n^2 \propto \omega_n\]

- 0: scattered waves
- 1: LH waves (4.6 GHz)
- 2: incident waves (60 GHz)
We can obtain a set of equations for mode amplitudes in the steady state weakly inhomogeneous plasma.

- Steady-state coupled mode equation by WKB techniques [3]

\[
\begin{align*}
\vec{v}_{g0} \cdot \frac{\partial}{\partial x} a_0 &= K a_1 a_2 \exp \int_{0}^{z} (-i) \kappa (x') dx' \\
\vec{v}_{g1} \cdot \frac{\partial}{\partial x} a_1 &= -K^* a_0 a_2^* \exp \int_{0}^{z} i \kappa (x') dx' \\
\vec{v}_{g2} \cdot \frac{\partial}{\partial x} a_2 &= -K^* a_0 a_1^* \exp \int_{0}^{z} i \kappa (x') dx'
\end{align*}
\]

- wave-number mismatch: \( \kappa = \vec{k}_0 - \vec{k}_1 - \vec{k}_2 \)

- coupling coefficient:

\[
K = \frac{-n_0 e^3}{m_e^2 \omega_{ce}^2} \frac{1}{\omega_0 \omega_2} |k_1| E_0^* E_1 E_2
\]

- Density fluctuations by LH waves

\[
|k_1|^2 E_1^2 = (\delta n)^2 \frac{m_e^2 \omega_{ce}^2}{n_0^2 e^2}
\]

- Assumption: LH mode to be pump wave

- Assume the mode amplitude of LH waves is unaffected by non-linear interactions, and remains constant

- The problem reduces to solve from three modes to two modes.
We formulate the problem and make assumptions in an attempt to obtain analytic solution.

- WKB variation of parameters (group velocity, coupling coefficients) remains small \([3]\)
  - Exponential factors have a significant effect in a distance \(L_d\) when approximately, \(\kappa' L^2_d \sim \pi\)
  - WKB assumption: \(k_{0,1,2} \gg 1/L\) where \(L\) is the scale length of inhomogeneity
  - \(L_d/L \sim \sqrt{\pi/k_{0,1,2}} L \ll 1\)
- We assume LH waves to be pump waves, as they are driven externally.
- We approximate the spatial variation of phase mismatch to be linear about the resonance point, \(x=0\):
  \[
  \kappa(x) \approx \left(\frac{d\kappa}{dx}\right)_{x=0} = \kappa' \equiv z
  \]
  \[
  \vec{v}_{g0} \cdot \frac{\partial}{\partial x} a_0 = K a_1 a_2 \exp(-i \kappa' x^2/2)
  \]
  \[
  \vec{v}_{g2} \cdot \frac{\partial}{\partial x} a_2 = -K^* a_0 a_1^* \exp(i \kappa' x^2/2)
  \]
Mode amplitudes of incoming ($a_2$) and scattered ($a_0$) waves are given in terms of parabolic cylinder function.

- **Normalization**
  \[ \xi \equiv \sqrt{\kappa'}/2 \cdot x \equiv x/l \]
  \[ l \equiv 2/\sqrt{\kappa'} \]

- The solution in terms of parabolic cylinder function [4,5].

\[
\begin{align*}
\frac{d}{d\xi} a_0 &= -l \frac{K}{v_{g0}} a_2 \exp(-i\xi^2) \\
\frac{d}{d\xi} a_2 &= -l \frac{K}{v_{g3}} a_0 \exp(i\xi^2)
\end{align*}
\]

\[
a_2 = A_1 e^{i\xi^2/2} D_{iZ/2}(\sqrt{2} e^{i\pi/4\xi})
\]

\[
a_0 = A_2 (-i) e^{i\pi/4 - i\xi^2/2} \frac{lK/v_{g0}}{\sqrt{2}} D_{iZ/2-1}(\sqrt{2} e^{i\pi/4\xi})
\]

\[ Z = l^2 K K^* / (v_{g0} v_{g2}) \]
Asymptotic expansion [4,5,6,7] of mode amplitudes can give us simple-looking solutions.

- Away from the resonance layer, we look for the asymptotic solutions to evaluate mode energy and power density in a simple-looking forms.
- Especially, we are interested in the mode amplitude at the far left of resonance layer to evaluate scattered power.

\[ a_2(\xi \to -\infty) \approx \exp\left(\frac{3\pi}{8}\right) \exp\left(i \frac{Z}{2} \ln(\sqrt{2} \xi)\right) \]

\[ a_0(\xi \to -\infty) \approx -i \exp i \frac{\pi}{4} \frac{lK/\nu g_0}{\sqrt{2}} \frac{\sqrt{2} \pi}{\Gamma(1-iZ/2)} \exp\left(\frac{\pi}{8} Z\right) \exp\left(-i \frac{Z}{2} \ln(\sqrt{2} |\xi|)\right) \]
We can finally then evaluate the scattered power.

- We can find scattered power density, \( S_0 \), from the ratio of mode amplitudes \( a_0 \) and \( a_2 \):
  \[
  \left( \frac{S_0}{S_2} \right)_{\xi \to -\infty} = \frac{v_{g0} \langle W_0 \rangle}{v_{g2} \langle W_2 \rangle}_{\xi \to -\infty} \approx \frac{\omega_0}{\omega_2} \frac{\pi l^2 K^2}{v_{g0}}
  \]

- Rewrite the coupling coefficient with the classical electron radius, \( r_e \):
  \[
  K^2 = 4 \pi^2 c^4 r_e^2 (\delta n)^2 \frac{1}{\omega_0 \omega_2}
  \]

- Ratio of scattering power to incident power in 1D geometry:
  \[
  \left( \frac{S_0}{S_2} \right)_{\xi \to -\infty} \approx \pi l^2 r_e^2 (\delta n)^2 \lambda_0^2
  \]

- Example
  - In linear density profile with density scale length, 3cm
    \[
    \kappa' \approx \frac{4000}{0.03} = 1.33 \times 10^5 \text{[m}^{-2}] \quad l = \sqrt{2/\kappa'} = 0.0039 \text{[m]}
    \]
    \[
    \lambda_0 \approx 0.005 \text{[m]}
    \]
    \[
    \delta n = 1 \times 10^{16} \text{[m}^{-3}] \quad \left( \frac{S_0}{S_2} \right)_{\xi \to -\infty} \approx -60 \text{dB}
    \]
Experimental results
We have upgraded the 60 GHz channel of reflectometer system in order to detect both frequency up- and down-shifted waves.

Hardware upgrade
- Gunn. Oscillators, Mixers
- intermediate frequency (IF) stages
- in-house built spectrum analyzer

Power Detector
- IF=1.1GHz
- waveform 1.0~1.2GHz
- Low pass filter 100kHz
- Logarithmic Amplifier
1-(1) For the first time, both frequency up-shifted and down-shifted scattered waves were detected simultaneously.

- Both up- and down-shifted scattered waves are mixed down to 1.1 GHz.
- The local oscillator (LO) sweeps around 1.1 GHz at 10~20 Hz.
- During LHRF injection, we observed the scattered waves.
- The increase of the noise floor is due to stronger background emission of O-mode electron cyclotron emission.
1-(2) Density perturbations of LH waves are measurable quantities.

- Scattered power is weak (~ -100 dBm), but measurable quantities.
- The power of incident waves leaving the antenna is ~+25dBm. The coupling loss due to the geometrical configuration (e.g. bi-static antenna system) is about 40 dB.
- Down-shifted scattered signals are weaker due to 3D effects.
1-(3) Scattered power is dependent on LH power. The width of scattered signals is broader than the width of incident waves.

- 600 kA, H-mode discharge
- $n_{\parallel} = 1.9$
- LH power = 200, 300, 500 kW
2-(1) In plasma with lower current compared to plasma with higher current, we observed stronger scattered power.

- No clear peak power was observed above the noise floor when the plasma current was above 700kA.
- Below 500kA, it is difficult to determine the current dependence of scattered power.
2-(2) In low plasma current, magnetic field lines between LH couplers and reflectometer antenna is more directly connected.

- Reflectometer system @ A-port midplane
- LH coupler @ C-port
- LH waves propagate along resonances, which follows magnetic field lines.
- At lower current, LH waves leaving the coupler are more likely to be located in the mid-plane in front of A-port.
2-(3) But in higher density plasma, we observed scattered power in plasma with higher current, mostly due to lower noise floor.

- In high density, background emission is reduced.
- Filter bandwidth: 100kHz
- No simple dependence on current is concluded.
3. Increase of background emission during LH experiment

- Background emission around 64.6 GHz increased by 0~20 dB, depending on plasma density.
- O-mode ECE is likely to cause background emission.
- The power of background emission is highly correlated to the X-mode electron cyclotron emission.
4-(1) Observation of LH waves in high density L-mode plasma

- The anomalous drop of LHCD efficiency in high density L-plasma [8,9]
- Discrepancy between experiments and simulations [8,9]
- 800kA, LH waves are accessible to the core of plasma
  - Noise floor decreases as plasma density increases.
4-(2) In higher density plasma, we observe less scattered power.

- Density profiles in front of LH coupler almost identical.
- 1D analytic model predicts less LH wave field exists.
- But, detailed trajectory of LH waves is not known.
- Other loss mechanisms in SOL can play a role.
  - Collisions
  - Ionization
  - Density fluctuations
4-(3) Both LH power and plasma density affects the scattered power.

- \( N || = 1.9 \)
- Bandpass filter: 100kHz
- Response of scattered power to LH power appears to be linear.
- In higher density, scattered power becomes weak.
4-(4) Scattered power was also dependent on the parallel refractive index of launched LH spectrum.

- Two different $n_{||}$ (1.9, 2.3) of LH waves were launched within in a single discharge.
- At $n_{||}=1.9$, we observed stronger peak power by about 50%.
Summary and Future work

- Model development for scattering of probe waves of reflectometer
  - Wave-number mismatch plays an role in the scattering process.
- Experimental results
  - Density fluctuations by LH waves were observed in SOL region.
  - Strong background emission can limit the observation of scattered signals.
  - Scattered power is observed to be dependent on plasma current, density, \( n_{||} \) and plasma mode.
  - The dependence of scattered on LH power is clear, but the density dependence is not yet resolved.
- Future work
  - Numerical simulation of the reflectometer response on LH waves will be conducted.
  - The role of low frequency density fluctuation on the propagation of LH waves will be assessed.
Reference


3-(2): Scattered power varies with the density, but it is not clear which part of LH waves interact with probe waves.

- Magnetic field lines are mostly below the mid-plane in front of A-port. In front of A-port, there are chances that LH waves are
  - directly from LH couplers
  - Undergone multiple passes before reach at mid-plane of A-port.
- Different SOL density profiles with and without LHRF in front of LH coupler
3-(3): Less scattered power is observed as density increases, but details needed to be considered to derive any meaningful conclusion.

Phase mismatch for three discharges are similar in front of LH coupler.

Then, 1D analytic model predicts

But, this is very crude estimation as the propagation and absorption of LH waves are sensitive to detailed information, for example, (1) exact density and magnetic profile, (2) the loss mechanisms of LHRF or (3) change of propagation due to density fluctuations.
Genray/CQL3D simulation
Density fluctuations by LH waves

- Cold plasma 1\textsuperscript{st} order analysis
  - Continuity equation
    \[ \delta n = \frac{k \cdot \dot{v}}{\omega} n_0 \]
  - Momentum equation
    \[ \frac{\delta n}{n_0} = \frac{i q}{m_e \omega^2} \left( E_x k_x \frac{\omega^2}{\omega^2 - \omega_{ce}^2} + k_z E_z \right) \]
  - Dispersion relation for electrostatic waves
    \[ \tan \theta = \frac{-P}{S} \]
    \[ \tan^2 \theta = \frac{k_x^2}{k_z^2} \approx \frac{E_x^2}{E_z^2} \gg 1 \]

- \[ \frac{\delta n}{n_0} \approx \frac{i q}{m_e \omega^2} \frac{\omega}{\omega_{pe}^2} |E| |k| \]
3-(3): With several assumptions, the power LH waves in front of A-port in higher density becomes weak.

- If we assume (1) that density profiles in front of A-port is similar to those in front of C-port when LH is off and (2) that probe waves directly interacts with LH waves leaving LH coupler, we can estimate that LH power is less in front of A-port in higher density plasma. (when compared 1101222016 to 110122014)
X-mode Michelson interferometer also indicated electron cyclotron (EC) emission around 64.6 GHz during LH experiment.

- Emission level measured from both X-mode and O-mode is highly correlated.
- But, emission from fast electrons in certain circumstances (high LH power, run-away electrons during disruption) should be examined.
- This emission makes difficult our measurement in low density plasma.
Scattering of O-mode waves can be understood in terms of non-linear mode-coupling.

- **Resonance condition** is satisfied in a single location due to spatial variation

\[
\vec{k}_0(x) = \vec{k}_1(x) \pm \vec{k}_2(x) \quad 0: \text{scattered waves}
\]

\[
\omega_0 = \omega_1 \pm \omega_2
\]

- **Two scattering interactions** in the presence of cut-off layer [1]

**Frequency up-shifted** scattering

- **Frequency down-shifted** scattering

![Graph showing frequency up-shifted and down-shifted scattering](image)
Coupled mode equations in 1D homogeneous medium

- Resonant condition \( \vec{k}_0 = \vec{k}_1 + \vec{k}_2 \), \( \omega_0 = \omega_1 + \omega_2 \)

- By combination of linear and non-linear version of Maxwell and momentum equations for a system conserving energy and momentum, we obtain the set of equations describing mode amplitudes.

\[
\left( \frac{\partial}{\partial t} + \vec{v}_{g0} \cdot \nabla \right) a_0 = K a_1 a_2 \\
\left( \frac{\partial}{\partial t} + \vec{v}_{g1} \cdot \nabla \right) a_1 = -K^* a_0 a_2^* \\
\left( \frac{\partial}{\partial t} + \vec{v}_{g2} \cdot \nabla \right) a_2 = -K^* a_0 a_1^*
\]

- Coupling coefficient is found from complex amplitudes of modes [1].

\[
K = \frac{-n_0 e^3}{m_e \omega_{ce}^2} \frac{1}{\omega_0 \omega_2} |k_1| E_0^* E_1 E_2 \\
E_0 = \sqrt{\frac{\omega_0}{2 \epsilon_0}} \\
E_2 = \sqrt{\frac{\omega_0}{2 \epsilon_0}}
\]

- Density fluctuation by LH waves is related to LH electric fields.

\[
|k_1|^2 E_1^2 = (\delta n)^2 \frac{m_e^2 \omega_{ce}^2}{n_0^2 e^2}
\]
1-(1): Measured scattered power (64.6 GHz) is dependent on the plasma current due to field line configuration.

- Reflectometer system @ A-port midplane
- LH coupler @ C-port
- LH waves propagate along resonances, which follows magnetic field lines.
- At lower current, LH waves leaving the coupler are more likely to be located in the mid-plane in front of A-port.
1-(2): Current dependence of scattered power suggests that probe waves (60 GHz) interact with LH waves (4.6 GHz) leaving the LH coupler.

- Plasma current varied from 300kA to 800kA.
- No clear scattered signals (above the noise floor) were observed when plasma current was above 700kA.
- Scattered power became stronger in lower plasma current.
- Accessibility of LH waves were on the border. (Launched n|| = 1.6)

![Graph showing scattered power vs. plasma current](image-url)
3-(3): Less scattered power is observed as density increases, but details needed to be considered to derive any meaningful conclusion.

Phase mismatch for three discharges are similar in front of LH coupler.

Then, 1D analytic model predicts

But, this is very crude estimation as the propagation and absorption of LH waves are sensitive to detailed information, for example, (1) exact density and magnetic profile, (2) the loss mechanisms of LHRF or (3) change of propagation due to density fluctuations.