Effect on plasma rotation of lower hybrid (LH) waves in Alcator C-Mod

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Lower hybrid waves drive current by injecting momentum toward the counter-current direction

- LH wave antenna is designed so that the waves propagate in $-\hat{b}$ direction → push electrons to $-\hat{b}$ by Landau resonance → drive current in $\hat{b}$
Ion toroidal rotation change is observed after injecting lower hybrid waves in Alcator C-Mod

- Acceleration in counter-current direction
- Different steady state rotation (High current vs. Low current)

![Graph showing ion rotation change](image)

<table>
<thead>
<tr>
<th>Ion rotation near r=0</th>
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<tbody>
<tr>
<td>Before LH wave</td>
</tr>
<tr>
<td>High $I_p$</td>
</tr>
<tr>
<td>Low $I_p$</td>
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</table>

Measured using X-ray crystal spectrometer in Alcator C-Mod by Y. Podpaly, J. Rice, M. Reinke, et al.
Relation between LH waves and ion toroidal rotation is complicated

LH wave

\[ \frac{dn}{dr}, \frac{dT}{dr}, \frac{dq}{dr} \]

Momentum Source

Ion toroidal rotation

Dispersion, Damping

\[ \frac{dn}{dr}, \frac{dT}{dr}, \frac{dq}{dr} \]

Current, Energy

\[ \frac{dn}{dr}, \frac{dT}{dr}, \frac{dq}{dr} \]

Viscosity

\[ \frac{dn}{dr}, \frac{dT}{dr}, \frac{dq}{dr} \]

Velocity shear

ITG, TEM

Transport

Turbulence
PART 1. EXTERNAL TORQUE
(TOROIDAL ANGULAR MOMENTUM TRANSFER FROM LOWER HYBRID WAVES TO IONS)
External torque and momentum radial transport determine ion rotation

- Ion toroidal angular momentum is determined by
  \[
  \frac{\partial}{\partial t} \langle n_i m_i R \dot{V}_\varphi \rangle_s = - \frac{1}{A} \frac{\partial}{\partial r} (A \Pi) + \frac{1}{c} \langle J_r B_\theta R \rangle_s + T_\varphi
  \]

  - \( \Pi = \) Radial transport of toroidal angular momentum
  - \( T_\varphi = \) External momentum torque from lower hybrid waves
  - Radial current vanishes due to ambipolarity

- Initial change of the rotation is determined by \( T_\varphi \)
  \[
  T_\varphi = \left\langle \int d^3 \nu \langle m_e R v_\varphi Q(f_e) \rangle_\alpha \right\rangle_s
  \]
  \[
  = \sum_k \left( \frac{\mathbf{k} \cdot \hat{\varphi}}{\omega} P_{abs,k} R \right)
  \]
Both perpendicular and parallel momentum are transferred from lower hybrid wave to electrons

- Full toroidal angular momentum of LH waves are transferred via both parallel and perpendicular momentum transfer.

- In previous research, the perpendicular momentum transfer has been ignored because of the gyro-averaged quasilinear diffusion operator:

\[
T_{\parallel}^\varphi = \left\langle \int d^3v \ m_e R v_{\varphi} \langle Q(f_e) \rangle_\alpha \right\rangle_s
\]

\[
\neq T_{\varphi} = \left\langle \int d^3v \langle m_e R v_{\varphi} Q(f_e) \rangle_\alpha \right\rangle_s
\]

\[
n/R = k_{\parallel} \cdot \varphi + k_{\perp} \cdot \varphi
\]
Various time scales for momentum transfer and transport

- A time-harmonic wave code of LH frequency (TORLH) → Evaluation of wave electric fields → Velocity space diffusion of electrons
- A Fokker-Planck code (CQL3D) → Evaluation of collisions → Steady state $\int_e T_\phi$ in $\tau_{LH \rightarrow e}^\perp$ → Momentum transfer to electrons $T_\phi$
- Momentum transfer to ions $T_\phi$ in $\tau_{\perp}^e \rightarrow i$ and $\tau_{\parallel}^e \rightarrow i$ [Lee (2012) PPCF]
- A gyrokinetic code (GS2) → Evaluation of turbulent radial momentum flux $\Pi$ → Rotation evolution in $\tau_{\phi}^{\text{transp}}$

Time Scale (Sec)

- $10^{-9}$
- $10^{-6}$
- $10^{-3}$
- $10^0$

RF Wave code

Fokker-Planck code

Gyrokinetic code

Lee (2012) PPCF
The wave code (TORLH) and the Fokker-Planck code (CQL3D) are iterated to find energy and momentum transfer.

- Iterations converge when two codes have self-consistent electric field and distribution function.

\[
E^i(\psi) = S_{TORLH} \left( Im \{ P(f^i_e(\psi, v)) \} \right)
\]

\[
f^{i+1}_e(\psi, v) = S_{CQL3D} \left( D_{ql}(E^i(\psi)) \right)
\]

In CQL3D [Harvey (1992) IAEA], \( f_e \) is obtained by balancing the quasilinear diffusion with collisions.

In TORLH, the quasilinear diffusion coefficient \( \propto E_z^2 \) is evaluated.
External torque is comparable with the experimentally observed acceleration

- External torque is important to explain the initial acceleration
- Observed acceleration profile is somewhat different from the torque profile
  - Radial momentum transport is already happening in $O(10)$ msec
  - Systematic errors of the observed radial profiles are not negligible
PART 2. MOMENTUM REDISTRIBUTION
(TURBULENT RADIAL TRANSPORT OF TOROIDAL ANGULAR MOMENTUM)
Ion toroidal angular momentum is redistributed due to turbulence

- Toroidal momentum transport equation is

$$\frac{\partial}{\partial t} \left< n_i m_i RV_\varphi \right>_s = -\frac{1}{A} \frac{\partial}{\partial r} (A\Pi) + T_\varphi$$

- The momentum redistribution is due to the turbulent radial flux of toroidal angular momentum

$$\Pi \simeq \left< m_i \int d^3 v f_i^{tb} (v \cdot \hat{\varphi} R)(v_E^{tb} \cdot \hat{r}) \right>_s$$

- Purpose of the study in this part: evaluate the momentum redistribution for non-rotating initial state

$$\Pi(\Omega_\varphi = 0)$$

- $$\Pi(\Omega_\varphi = 0) > 0$$ expels momentum $$\Rightarrow$$ $$\partial\Omega_\varphi / \partial t < 0$$ (counter-current)
- $$\Pi(\Omega_\varphi = 0) < 0$$ brings momentum $$\Rightarrow$$ $$\partial\Omega_\varphi / \partial t > 0$$ (co-current)
Symmetric turbulence cannot result in momentum transport

- Symmetry in gyrokinetic-Maxwell equations without formally small effects [Sugama (2011) and Parra (2011)]

\[ \Pi_0(\Omega_\phi = 0) = \sum_{k_r} \int d\theta \pi_0(\theta, v_\parallel, k_r) = 0 \]

\[ \pi_0(\theta, v_\parallel, k_r) = -\pi_0(-\theta, -v_\parallel, -k_r) \]
What are the perturbations that break the symmetry?

- Up-down asymmetric magnetic equilibrium [Camenen (2009)]

- Slow variation of radial profile (Global codes) [Wang (2009) and Waltz (2011)]

- Slow poloidal variation of turbulence [Barnes (2011) and Sung (2013)]

- Small deviation from Maxwellian equilibria (diamagnetic effect) [Lee (2013) and Barnes (2013)]
Can diamagnetic flows induce intrinsic rotation?

- Ion toroidal rotation in a tokamak is

\[ V_\varphi = \Omega_\varphi R = (\Omega_\varphi,d + \Omega_\varphi,E)R \]

- For non-rotating state in which the diamagnetic flow and the ExB flow cancel each other, finite momentum flux occurs

\[ \Omega_\varphi = \Omega_\varphi,d + \Omega_\varphi,E = 0 \quad \text{diamagnetic ExB flow} \]

\[ \Pi(\Omega_\varphi = 0) \neq 0 \]

\[ \Omega_\varphi,d = -\frac{1}{B_\theta R} \frac{c}{Z e n_i} \left( \frac{\partial p_i}{\partial r} + C_T(\theta) \frac{\partial T_i}{\partial r} \right) \quad \text{due to pressure and temperature gradient} \]

\[ \Omega_\varphi,E = \frac{c}{R} \frac{E_r}{B_\theta} \quad \text{due to radial electric field} \]
Gyrokinetic equations are modified differently by diamagnetic flow \((\Omega_{\varphi,d})\) and ExB flow \((\Omega_{\varphi,E})\)

- For low flow (Mach~ 0.1) without external torque or only with LH waves
  \[
  \Omega_{\varphi,E} \sim \Omega_{\varphi,d} \sim \left(\frac{B}{B_\theta}\right)\left(\frac{\rho_i}{L_T}\right)v_{ti}
  \]

- Poloidal rhostar \(\left(\frac{B}{B_\theta}\right)\left(\frac{\rho_i}{L_T}\right)\) higher order correction to gyrokinetic equation in the lab frame is

\[
\frac{\partial f^{tb}}{\partial t} + \left( v_\parallel \hat{b} + v_M + (v_{E0} + v_{tb}^{E}) \right) \cdot \nabla f^{tb} - \frac{Ze}{m_i} v_M \cdot \nabla \phi_0 \frac{\partial f^{tb}}{\partial E} = -v_{tb}^{E} \cdot \nabla (F_0 + F_1) + \frac{Ze}{m_i} [v_\parallel \hat{b} + v_M] \cdot \nabla \langle \phi^{tb} \rangle \frac{\partial (F_0 + F_1)}{\partial E} + \langle C(f) \rangle
\]

where \(F_0 = F_M (v - \Omega_{\varphi,E} R \hat{\phi})\) is the shifted Maxwellian only by ExB flow,

and \(F_1 \left( \frac{\partial p}{\partial r}, \frac{\partial T}{\partial r}, \nu_\ast, \ldots \right) = F^{\Omega_{\varphi,d}}_1 + F^{\text{other}}_1\) includes

the diamagnetic flow \(F^{\Omega_{\varphi,d}}_1 = \frac{B_\varphi}{B} \frac{m_i v_\parallel}{T_i} \Omega_{\varphi,d} R F_M\)
The turbulent radial momentum transport can be linearized and split into momentum pinch, diffusion and intrinsic momentum flux

- The momentum redistribution is due to the turbulent radial flux of toroidal angular momentum

\[
\Pi \simeq \left\langle m_i \int d^3 v f_i^{tb}(v \cdot \hat{\varphi} R)(v_E^{tb} \cdot \hat{r}) \right\rangle_s
\]

- Each contribution of flow and radial flow shear to the turbulent radial flux can be linearized for low flow

\[
\Pi \left( \Omega_\varphi, \frac{\partial \Omega_\varphi}{\partial r}, \ldots \right) \simeq \Pi_{\text{int}} - P_\varphi n_i m_i R^2 \Omega_\varphi - \chi_\varphi n_i m_i R^2 \frac{\partial \Omega_\varphi}{\partial r}
\]
Different transport for diamagnetic flow and ExB flow results in intrinsic momentum transport

- For both diamagnetic flow and ExB flow, 
  \[ \Omega_\varphi = \Omega_\varphi,d + \Omega_\varphi,E \]
  
  \[ \Pi \left( \Omega_\varphi,d, \Omega_\varphi,E, \frac{\partial \Omega_\varphi,d}{\partial r}, \frac{\partial \Omega_\varphi,E}{\partial r}, \ldots \right) \]
  
  \[ \approx \Pi_{other} - n_i m_i R^2 [P_\varphi,d \Omega_\varphi,d + P_\varphi,E \Omega_\varphi,E] - n_i m_i R^2 \left[ \chi_\varphi,d \frac{\partial \Omega_\varphi,d}{\partial r} + \chi_\varphi,E \frac{\partial \Omega_\varphi,E}{\partial r} \right] \]

- For the diamagnetic flow and ExB flow canceling each other,
  
  \[ \Omega_\varphi = \Omega_\varphi,d + \Omega_\varphi,E = 0 \quad \text{and} \quad \frac{\partial \Omega_\varphi}{\partial r} = \frac{\partial \Omega_\varphi,d}{\partial r} + \frac{\partial \Omega_\varphi,E}{\partial r} = 0 \]

  \[ \Pi_{\text{int}} = \Pi_{other} - n_i m_i R^2 \Delta P_\varphi \Omega_\varphi,d - n_i m_i R^2 \Delta \chi_\varphi \left( \frac{\partial \Omega_\varphi,d}{\partial r} \right) \]

  where \( \Delta P_\varphi = P_\varphi,d - P_\varphi,E \) and \( \Delta \chi_\varphi = \chi_\varphi,d - \chi_\varphi,E \)
PART 3. CHANGE IN STEADY STATE ROTATION DUE TO LOWER HYBRID WAVES

(COMPARISON OF THEORY WITH EXPERIMENTAL RESULTS)

- LH wave
  - Current, Energy
  - $dT/dr, dq/dr$
  - Part 3
  - ITG, TEM

- Torque

- Ion rotation

- Viscosity

- Turbulence
Steady state rotation is reconstructed by balance between external torque and radial momentum transport

- Assume zero intrinsic flux to see the effects of pinch and diffusion

\[- \frac{1}{A} \frac{\partial}{\partial r} (A \Pi) + T_\varphi = 0 \quad \text{where} \quad \Pi \approx \Pi_{\text{int}} - P_\varphi n_i m_i R^2 \Omega_\varphi - \chi_\varphi n_i m_i R^2 \frac{\partial \Omega_\varphi}{\partial r} \neq 0 \]

- Reconstruct rotation radial profile.

\[
\Omega_\varphi(r) = \Omega_\varphi(a) \exp \left( \int_a^r \frac{P_\varphi}{\chi_\varphi} dr' \right) + \int_r^a \frac{\int_0^{r'} dr'' A(r'') T_\varphi(r'')} {A \chi_\varphi n_i m_i \langle R^2 \rangle_s} \exp \left( \int_r^{r'} \frac{P_\varphi}{\chi_\varphi} dr'' \right)
\]

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<thead>
<tr>
<th></th>
<th>GS2</th>
<th>TRANSP</th>
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<tbody>
<tr>
<td></td>
<td>Pr = \chi_\varphi / \chi_i</td>
<td>(P_\varphi / \chi_\varphi) R_0</td>
</tr>
<tr>
<td>(1) inner radius (r/a=0.37)</td>
<td>0.77</td>
<td>1.15</td>
</tr>
<tr>
<td>(2) mid-radius (r/a=0.54)</td>
<td>0.60</td>
<td>2.06</td>
</tr>
<tr>
<td>(3) outer radius (r/a=0.79)</td>
<td>0.85</td>
<td>2.73</td>
</tr>
</tbody>
</table>
Rotation change by external torque without intrinsic momentum flux is comparable to the observed change in the counter-current

- For high plasma current case ($I_p=700$ kA)

- The change of the observed core rotation due to lower hybrid wave is similar to the estimation using the simulated torque, pinch and diffusion

- Rapid spatial oscillation in observed rotation for $r/a > 0.5$ may be unphysical due to the weak radiation signal and the inversion errors of the line integrated signal.
Rotation change by external torque without intrinsic momentum flux is comparable to the observed change in the counter-current.

- For low plasma current case ($I_p=350$ kA)

- Intrinsic momentum flux is required to explain the co-current change from $150$ msec to $450$ msec after LH wave injection.
For no external torque, the intrinsic momentum flux is:

$$\frac{\Pi_{\text{int}}}{Q_i} = - \frac{\Pi_{\text{pinch}}}{Q_i} - \frac{\Pi_{\text{diff}}}{Q_i}$$

Use experimental radial profiles of ion rotation and temperature to estimate momentum pinch and diffusion.

- With full radial profiles of rotation vs. Two values at the center and the edge.

For high current case:

For low current case:

- Positive momentum flux means counter-current rotation.

For high current case:

For low current case:

Positive momentum flux means counter-current rotation.
Intrinsic momentum fluxes due to diamagnetic flow are simulated using experimental plasma parameters

- Theory can explain why only low current case shows different rotation change due to intrinsic momentum flux

\[
\Omega_{\varphi,d} \propto q \propto 1/I_p
\]

<table>
<thead>
<tr>
<th></th>
<th>NEO</th>
<th>GS2</th>
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<tbody>
<tr>
<td></td>
<td>$\frac{\Omega_{\varphi,d} R_0}{v_{ti}}$</td>
<td>$\frac{\vartheta \Omega_{\varphi,d} R_0 a}{\partial r v_{ti}}$</td>
</tr>
<tr>
<td>High current</td>
<td>before LH</td>
<td>0.058</td>
</tr>
<tr>
<td></td>
<td>after LH</td>
<td>0.054</td>
</tr>
<tr>
<td>Low current</td>
<td>before LH</td>
<td>0.140</td>
</tr>
<tr>
<td></td>
<td>after LH</td>
<td>0.171</td>
</tr>
</tbody>
</table>

Larger $\Pi_{\text{int}}$ can give larger counter-current rotation (consistent with experiments)
Intrinsic momentum fluxes due to diamagnetic flow are simulated using experimental plasma parameters

- Theory can explain why only low current case shows different rotation change due to intrinsic momentum flux

<table>
<thead>
<tr>
<th></th>
<th>NEO</th>
<th>GS2</th>
</tr>
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<tbody>
<tr>
<td></td>
<td>$\frac{\Omega_{d,R_0}}{v_{ti}}$</td>
<td>$\frac{\delta\Omega_{d,R_0a}}{\partial r}v_{ti}$</td>
</tr>
<tr>
<td><strong>High current</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>before LH</td>
<td>0.058</td>
<td>-0.028</td>
</tr>
<tr>
<td>after LH</td>
<td>0.054</td>
<td>-0.001</td>
</tr>
<tr>
<td><strong>Low current</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>before LH</td>
<td><strong>0.140</strong></td>
<td><strong>-0.151</strong></td>
</tr>
<tr>
<td>after LH</td>
<td><strong>0.171</strong></td>
<td><strong>-0.391</strong></td>
</tr>
</tbody>
</table>

Larger $\Pi_{int}$ can give larger counter-current rotation (inconsistent with experiments)

- Theory cannot produce the reversed direction of rotation change (which requires reduction of the positive flux) after LH wave injection in low current case→ Try the different experimental plasma parameters within error bars
Sensitivity analysis (1): ion temperature profile

- Ion temperature gradient change by ±20% result in the change of the intrinsic momentum flux by 31%

- A significant change in second radial derivative of ion temperature (about 7 times) reduces the intrinsic momentum flux only by 20%

→ Intrinsic momentum flux may not be significantly changed by ion temperature profiles
Sensitivity analysis (2): plasma current profile

- Reduced magnetic shear decreases the diamagnetic flow shear, and therefore it changes the momentum fluxes.

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<thead>
<tr>
<th></th>
<th>Inputs</th>
<th>NEO</th>
<th>GS2</th>
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<tbody>
<tr>
<td></td>
<td></td>
<td>$\frac{\Omega_{\varphi,d} R_0}{v_{ti}}$</td>
<td>$\frac{\partial \Omega_{\varphi,d} R_0}{\partial r} \frac{1}{v_{ti}}$</td>
</tr>
<tr>
<td>Low current case</td>
<td>$\hat{s}=2.11$</td>
<td>0.171</td>
<td>-0.391</td>
</tr>
<tr>
<td>after LH wave</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>25% decrease</td>
<td>$\hat{s}=1.57$</td>
<td>0.171</td>
<td>-0.499</td>
</tr>
<tr>
<td>50% decrease</td>
<td>$\hat{s}=1.05$</td>
<td>0.171</td>
<td>-0.614</td>
</tr>
</tbody>
</table>

Smaller $\Pi_{\text{int}}$ can give reduced counter-current rotation (consistent with experiments).

- The reduction in counter-current rotation due to $\Delta \left( \frac{\Pi_{\text{int}}}{Q_i} \right) \frac{v_{ti}}{R_0} = -0.066$ is not sufficient to dominate over the increase in counter-current rotation by the external torque.

$$\Delta \Omega_{\varphi} \sim -a \frac{\Delta \Pi_{\text{int}}}{\chi_{\varphi} n_i m_i \langle R^2 \rangle_s} = 46 \text{kHz} \quad \text{vs.} \quad \Delta \Omega_{\varphi} \sim a \frac{\int_0^r dr'' A(r'') T_{\varphi}(r'')} {A \chi_{\varphi} n_i m_i \langle R^2 \rangle_s} = -52 \text{kHz}$$
Plasma current radial profile changes due to LH wave current drive

- Safety factor increase generally, and magnetic shear changes significantly depending on the location of LH wave power absorption.

- Reduced magnetic shear can change the momentum flux, which may result in the different direction of the rotation change [Lee, MIT Thesis (2013)].

- The change in the current profile due to the lower hybrid waves is in a current resistive relaxation time scale $O(100)$ msec, in which the reversal of the rotation change is observed.
Some other mechanisms proposed appear too weak to explain the co-current rotation change

- The momentum carried by the parasitic co-current propagating waves is small in the core of tokamak.

- The effect of trapped electrons on the momentum transfer is negligible: The population of resonant trapped electrons is much smaller (below 1%) than that by resonant passing electrons due to high phase velocity of the waves.

- A transient change in the radial electric field due to non-ambipolar electron radial drift cannot be directly related to the toroidal rotation change after an ion-ion collision time in which the poloidal rotation decay.
Conclusions

• The parallel and perpendicular components of the toroidal angular momentum are transferred from the waves to ions through electrons via two different channels.

• After ion collision time, the external torque to ions is the same as the injected toroidal angular momentum by LH waves, and it is comparable to the initial acceleration observed in Alcator C-Mod.

• The momentum transferred to the ions is transported out by turbulent radial transport (momentum pinch, momentum diffusion, and intrinsic momentum flux).

• The change in steady state rotation by the external torque without intrinsic momentum flux is comparable to the observed change in the counter-current direction.
Conclusions

• The intrinsic momentum flux due to diamagnetic flow effects can explain the size and the direction of the intrinsic rotation before LH wave injection.

• The diamagnetic flow is proportional to the inverse of plasma current, and it may explain why only low current case shows different rotation change.

• The intrinsic momentum flux using experimental parameters cannot reproduce the reversal of rotation change. The reversal may be partially explained by a feature of intrinsic momentum flux depending on the magnetic shear.

• This comparison between theory and experiments is preliminary. For better comparison, the precision of the radial profile of several plasma parameters must to be improved and the other important effects on momentum transport need to be investigated theoretically.
References


• External torque evaluation:  J. P. Lee et. al. 19th RF conference proceedings 1406 (2011) 459, J. P. Lee et. al. PPCF 54 (2012) 125005


• Comparison of theory with observed rotation: J. P. Lee, MIT Thesis (2013)