On the formation and stability of impurity ‘snakes’ in tokamak plasmas

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Outline

① Background and motivation

① Typical SXR observations and full diagnostic suite

② Snake formation and theoretical implications.

③ Summary
Background

蛇的震荡特点是通过一个局部化和增强的等离子密度区域，该区域旋转在各种诊断的视野内，且集中在或位于$q=1$表面。


② ③ $D_2$ 颗粒诱导的蛇的震荡最初在JET (Weller, PRL ‘87) 被观测，作为在理学$q$-表面的持续密度扰动。

④ 标准模型表明，
- 局部冷却$q=1$表面。
- 增加局部等离子体电阻率。
- 导致电流密度下降。
- 引起岛的形成。
- 岛在锯齿中幸存。

A. Weller  
PRL ‘87
Motivation for new studies of *snakes*

1. The formation of the *snake*, the reasons for its improved stability (e.g. surviving multiple sawtooth crashes) and its impact on the background plasma are still unknown.

2. New diagnostics with adequate spatial resolution can be used for the first time to study the role of $P_{\text{rad}}$, $n_z$, $v_{e,i}$, $\eta$, $\nu_\phi$, $p=nT$ and $Z_{\text{eff}}$.

3. Simulations of ideal-MHD equilibrium states have reproduced the ‘original’ snakes observed at JET (Cooper, PRL’10, NF’11) and predicted the helical ITER equilibria (Cooper, PPCF’11).

4. There are growing concerns that these 3D structures (e.g. triggered by particle fueling or impurity accumulation) could enhance the redistribution of fast ions ($r_{q=1,\text{ITER}}\sim 1m$).
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(1,1) snake modes in C-Mod are routinely observed using the x-ray tomographic arrays.

1. Observed during the current ramp-up phase or early in the plasma current flattop.
2. The high-Te,edge increases high-Z impurity erosion ⇒ allow on-axis impurity peaking.
Suite of novel diagnostics installed in Alcator C-Mod enables new snake 3D studies

SXR tomographic System (XTOMO)
McPherson spectrometer suggests that Mo is the main high-Z impurity before snake formation.

1. Typical enhancement of Mo-line emission from core accumulation is ×4-10

2. Lack of emission from stainless steel constituents
Almost all the ionized Mo in the core of Alcator C-Mod is Ne-like Mo (Mo^{32+})

① Mo is the main high-Z intrinsic impurity at C-Mod (PFCs).

② The average Mo ion charge for core electron temperature of interest is $\langle Z \rangle \sim 32$.

③ Data from XTOMO and AXUV arrays - with known cooling factors - can be used as a proxy for $n_{Mo}/n_e$:

$$L_{Mo} \sim 7 \times 10^{-32} \text{ W} \cdot \text{m}^3$$
High resolution x-ray imaging spectrometer monitors He-like and H-like Ar, as well as Ne-like molybdenum (Mo$^{32+}$).
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Similar systems have been installed in NSTX, LHD, KSTAR, and EAST.
Molybdenum from the metal walls is responsible for the enhanced SXR and $P_{\text{rad}}$.

Brightness of $\text{Mo}^{32+}$ peaking at the core before "\text{snake}" is observed.
Enhanced SXR brightness is well correlated with an increase in Mo$^{32+}$ line intensity. Conclude that the “snake-like” pattern in the brightness data is formed by a small region of localized and enhanced molybdenum density.
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In few words: Snake formation can be considered as a multi-stage process.

- a) $\Delta t \sim 16 \text{ ms}$ (during impurity accumulation)
- b) $\Delta t \sim 0 \text{ ms}$ (before kink)
- c) $\Delta t \sim 0.5 \text{ ms}$ (kink-like)

Circular Displaced core Rotating kink-like perturbation

Sawtooth-free
2D SXR tomography of snake formation resembles a kink-like impurity density perturbation ($\delta \varepsilon \sim n_e \delta n_{Mo}$).
High-resolution AXUV and ECE measurements suggest the presence of $n_Z$ and $T_e$ fluctuations during snake formation.

- **a)** $P_{rad}$, AXUV - Array A

- **b)** $T_{e0}$, FRC-ECE

- **c)** Arrays A vs J
  \[ \Delta \phi_{A,AJ} \sim 105^\circ \]

- **d)**
  \[ dT_e/T_e \sim 1.2\% \]
  
  - $R \sim 70.1$ cm
  - $R \sim 69.0$ cm
  - $R \sim 68.6$ cm
High-resolution AXUV arrays enable $P_{\text{rad}}$ estimates before and during snake formation.

AXUV-$P_{\text{rad}}$ observations show the $n_{\text{Mo}}$ “without” contamination by $T_e$. 
High-resolution AXUV arrays enable $P_{\text{rad}}$ estimates before and during snake formation.

1. $P_{\text{rad}}$ estimates before snake:

\[ \Delta P_{\text{rad}} \sim n_e n_{\text{Mo}} L_{\text{Mo}} \]

$L_{\text{Mo}} \sim 7 \times 10^{-32} \text{ W} \cdot \text{m}^3$

$n_{e,0} \sim 2.2 \times 10^{20} \text{ m}^{-3}$

$\Delta P_{\text{rad},0} \sim 2.1 \text{ MW/m}^3$

$\Rightarrow n_{\text{Mo}} \sim 1.4 \times 10^{17} \text{ m}^{-3}$

$(c_{\text{Mo}} \sim 6 \times 10^{-4})$

2. $P_{\text{rad}}$ during kinked-snake:

$n_{e,0} \sim 1.7 \times 10^{20} \text{ m}^{-3}$

$\Delta P_{\text{rad},0} \sim 2.7 \text{ MW/m}^3$

$\Rightarrow n_{\text{Mo}} \sim 2.2 \times 10^{17} \text{ m}^{-3}$

$(c_{\text{Mo}} \sim 13 \times 10^{-4})$
Sawtooth-free kinked-snake configurations resembles tokamak equilibria that have helical pressure distributions.

Ideal MHD equilibria with reversed $q$-profiles [Cooper, PRL, (2010)] can have non-axisymmetric pressure profiles without islands.

Bifurcated helical equilibrium states resemble saturated internal ideal kink mode structures like early C-Mod snake formation.
But, the last stage resembles that of a magnetic island produced by a resistive 1/1 internal kink.
Later snake: SXR tomographic reconstruction shows 1/1 crescent shape

① Dimensions: \( r_s \sim 5 \text{ cm} \), full-width \( \omega_{sat} \sim 6 \text{ cm} \) and \( \pi < \sigma_\theta < 5\pi/4 \) radians.

① Perturbation travels in the electron diamagnetic drift direction.

② From the transit time \( \tau \sim 185-200 \mu \text{s} \) we calculate \( v_\phi \sim 21-23 \text{ km/s} \), which agrees with the \( v_\phi \) measured from the x-ray line Doppler shifts.
High-Z impurity snakes are surprisingly resilient to sawtooth crashes.

1. \( \tau_{\text{snake}} \approx 185-200 \ \mu s \) while \( \tau_{\text{sawtooth}} \approx 3.66 \ \text{ms} \)

\[ \Rightarrow \sim 20 \ \text{snake periods}. \]

2. Crashes may cause a transient reduction of \( r_{q=1} \)

\( \delta r_1 \lesssim 1 \ \text{cm} \).
Circular core moves radially outwards during sawtooth crash.

① During the crash the small core moves rapidly outwards to the edge of the crescent radius.

② Snake density is nearly “untouched” by thermal crash + heat pulse.

③ The location of the peak emission shrinks inward by 1-2 cm at the crash.

④ The snake toroidal transit frequency slows by $\sim 25\%$. 
Both sawtooth crashes & background transport contribute in flushing impurities from snake

① SXR emissivity decreased ~1.0 MW/m³ in 50 ms.

② Circular core displaced in time back to the original magnetic axis.

③ \( r_q=1 \) might also broaden in time due to current density penetration.
High-resolution AXUV reconstructions of $\delta n_Z$ from $\delta P_{\text{rad}}$ show strong impurity concentration

Estimate:

$$\delta n_{Mo} \sim \frac{\delta P_{\text{rad}}}{n_{e,0} L_{Mo}}$$

Do $\delta P_{\text{rad}}$, $\delta n_Z$, & $\delta Z_{\text{eff}}$ have an effect on the formation and stability of the mode?
Theoretical implications: cannot use a standard (Wesson-like) MRE model

1. The injection of impurities and cooling of flux surfaces does not constitute a necessary condition for the impurity-snake formation.

2. Extended version of the MRE greatly over-emphasizes the local effects of the $P_{\text{rad}}$ and $Z_{\text{eff}}$ (e.g. saturation $\Rightarrow \Delta' < 0$). The $\Delta'$ matching is unrealistic since the mode displaces the entire core.

3. Observation of internal kink-mode in the sawtooth-free phase (not an island!).

4. Models of the snake based on pressure balance have a fundamental difficulty in explaining its co-existence with periodic sawtooth crashes.
Can we conceive obtaining a $\Delta'$ for the (1,1)?

- $R_0 \sim 67-68$ cm
- $\omega_{\text{sat}} \sim 1-1.5$ cm
- $r_s \sim 5$ cm
- $P_{\text{rad,BOLO}} (\theta \sim 0, t=0.340$ s)
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Dynamic snake with density and temperature

1. MHD simulations with separate temperature and density evolution find a new nonlinear $m/n=1/1$ kink-like mode compatible with C-Mod early snake.

2. Quasi-steady-state $m/n=1/1$ helical density perturbation can exist over $q \gtrapprox 1$.

3. Inside $q<1$, the internal kink-like mode tends to minimize the 1/1 pressure gradient, while $n$ and $T$ are compatible with sawtooth crashes.


Summary

① A suite of novel imaging diagnostics enables estimates of the \( n = 1 \) helical structure of \( P_{\text{rad}}, n_z, n_e, T_e, Z_{\text{eff}}, \) and \( \eta \) inside the \( q \leq 1 \) region with adequate temporal and spatial resolution.

② The unintentional or deliberate injection of impurities with the subsequent cooling of the flux surfaces, does not constitute a necessary condition for the impurity-snake formation.

① The condition for a saturated island can not be inferred from an extended Modified Rutherford formalism, but from considering 3D ideal or resistive quasi-steady-state kinked equilibria.

① These new high-resolution observations show details of the snake evolution and the accompanying sawtooth oscillations that suggest important differences between the density and temperature dynamics, ruling out a purely pressure-driven process.

② The observed differences are consistent with the results of nonlinear M3D MHD simulations that evolve plasma density and temperature separately.
EXTRA
3D equilibria can contain 1/1 surfaces

A. Stabilizing term due to reversed axisymmetric dp/dr appears in cylindrical and toroidal linear theory:

Shafranov: \[ \gamma_{m=1} = \int_0^{r_s} \left( -2e^2r^2 \frac{\partial P}{\partial r} + e^2rB^2_\theta (3q + 1)(1 - q) \right) dr \]

Bussac: \[ \delta W^{(1,1)} \approx \frac{3r_1^4}{R_0^4} (1 - q_0) \left( \beta^2_{\theta,c} - \hat{\beta}^2_{\theta} \right) \]

\[ \hat{\beta}_\theta = \frac{-2\mu_0}{B^2_\theta(r_1)r_1^2} \int_0^{r_1} r^2 \frac{dp}{dr} dr \]

B. Park et al. [PoF, (1987)], showed the existence of a saturated island if the pressure inside the magnetic island is higher than in the plasma core.

C. Thyagaraja [PoF, (1991)] showed the existence of an equilibrium MHD island for \( q_0 < 1 \). With a flat pressure profile, persistence of the 1/1 island requires non uniform resistivity within the island!

\[ \Delta \eta \eta_0 > 0 \]