Stabilization of Electron-Scale Turbulence by Electron Density Gradient in NSTX

J. Ruiz Ruiz\textsuperscript{1}
Y. Ren\textsuperscript{2}, W. Guttenfelder\textsuperscript{2}, A. E. White\textsuperscript{1},
S.M. Kaye\textsuperscript{2}, B. P. LeBlanc\textsuperscript{2}, E. Mazzucato\textsuperscript{2}, K.C. Lee\textsuperscript{3},
C.W. Domier\textsuperscript{4}, D. R. Smith\textsuperscript{5}, H. Yuh\textsuperscript{6}


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Anomalous Electron Thermal Transport is Observed in All NSTX Confinement Regimes

• **Transport of electron energy** in most tokamak experiments is observed to exceed predictions of neoclassical theory.

• Theory and experiments suggest that **ETG turbulence** is a candidate for anomalous electron thermal transport in some operating regimes.

• A *microwave collective scattering diagnostic* is used at NSTX to measure electron-scale density fluctuations indicative of **high-k turbulence** \((k \rho_s > 1)\).
Critical Gradient and Critical ETG Formula

- Normalized gradient of quantity $X$

\[ \frac{R}{L_X} = -R \left( \nabla X / X \right) \]

- Critical gradient

\[ q_{r\, turb} = \chi_{GB} f(\hat{s}, q, \nabla n_e, ...) \left( \frac{R}{L_{Te}} - \left( \frac{R}{L_{Te}} \right)_c \right) \]

Nonlinear dependence  Linear threshold


\[ (\frac{R}{L_{Te}})_{crit} = \max \left\{ \frac{0.8R}{L_{ne}}, \left( 1 + \tau \right)(1.33 + 1.91\hat{s} / q)(1 - 1.5\varepsilon)(1 + 0.3\varepsilon \, d\kappa / d\varepsilon) \right\} \]

with \[ \tau = Z_{eff} \frac{T_e}{T_i} \]

Applicability: low-$\beta$, positive $\hat{s}$ and large aspect ratio with local Miller equilibrium (Miller et al PoP 1998).
Previous Work

• First direct experimental demonstration of density gradient stabilization of e⁻-scale turbulence (*Ren et al. PRL 2011*). Shot 140620.
  - **ELM event** at t~525 ms ➔ change in density gradient.
  - Stabilization of lower-k e⁻-scale fluctuations ($k_\perp \rho_s < 10$).

• Nonlinear gyrokinetic simulations show the effect of density gradient on transport (*Ren et al. PoP 2012*). Shot 140620.

• Here, I focus on the effect of density gradient on e⁻-scale fluctuations and on the ETG unstable wavenumbers on shot 141767.
Collective Scattering is Used to Measure High-k Turbulence

- Collective/coherent scattering
  \[ k \lambda_D \leq 1 \]

- Scattered power density
  \[ \frac{d^2 P}{d\Omega d\nu} = P_i r_e^2 L_z |\Pi \cdot \hat{e}|^2 \left| \frac{\tilde{n}_e(k,\omega)}{V T} \right|^2 \]

- **Three wave-coupling** between incident beam \((k_i, \omega_i)\) and plasma \((k, \omega)\)
  \[ \vec{k}_s = \vec{k} + \vec{k}_i \quad \omega_s = \omega + \omega_i \]

- \(\omega_i, \omega_s \gg \omega\) imposes Bragg condition
  \[ k = 2k_i \sin(\theta_s/2) \]
High-k Microwave Scattering Diagnostic at NSTX

- Gaussian Probe beam: 15 mW, 280 GHz, $\lambda_i \sim 1.07$ mm, $a = 3$ cm (1/e² radius).
- Propagation close to midplane => $k_r$ spectrum.
- 5 detection channels => range $k_r \sim 5$-30 cm⁻¹ (*high-k*).
- Wavenumber resolution $\Delta k = \pm 0.7$ cm⁻¹.
- Radial coverage: $R = 106$-144 cm.
- Radial resolution: $\Delta R = \pm 2$ cm (unique feature).

View from top of NSTX (D.R. Smith PhD thesis 2009)
Each Channel of the NSTX High-k Scattering System Detects a Fluctuation Wavenumber $k$

- Channel 1 detects highest $k_\perp$ and $k_t$, Doppler shift is greatest ($f_D = k_t v_t / 2\pi$).
- High peak at $f \sim 0$ corresponds to stray radiation.
- Scattering region $R \sim 135-136$ cm, $r/a \sim 0.7-0.8$. (major radius 0.85 m, minor radius 0.68 m).

Shot 141767
A Set of NBI-heated H-mode Plasmas is Used to Study High-k Turbulence during Current Ramp-down

- **NBI heated**, HHFW heating is absent during the run.

- **Controlled Current ramp down** between $t = 400$ ms and $t = 450$ ms (from LRDFIT).

- Time range of interest is $t >\sim 300$ ms, covering current ramp-down phase, and after ELM event at $t \sim 290$ ms.

- **MHD activity is quiet during time range of interest.** *(cf. low-f Mirnov signal).*

- Line integrated density is fairly constant during the time range of interest.
Observed High-k Fluctuations Correlate to Local Electron Density Gradient

\[ \nabla T_e \rightarrow \textbf{Drives} \text{ ETG} \]

\[ \nabla n_e \rightarrow \textbf{Stabilizes} \text{ ETG} \]

Two competing effects: \( \nabla n_e \) is dominant effect.
Theory Predicts that Electron Density Gradient Can Affect the Difference $(R/L_{Te})_c - R/L_{Te}$ and Stabilize Turbulence

- Jenko critical gradient is a maximum of a $R/L_{ne}$ term and an $s/q$ term.

$$ (R/L_{Te})_{crit} = \max \left\{ \begin{array}{c} 0.8 R/L_{ne} \\ (1+\tau)(1.33+1.91\hat{s}/q)(1-1.5\varepsilon)(1+0.3\varepsilon \frac{d\kappa}{d\varepsilon}) \end{array} \right\} \quad \text{with} \quad \tau = Z_{eff} \frac{T_e}{T_i} $$

- Higher values of $R/L_{ne}$ raise the critical gradient for ETG (possibly above the experimental gradient value). This should have a stabilizing effect on turbulence.
Observed Fluctuation Amplitude Correlates to Difference Between Critical and Experimental Temperature Gradient

- Total scattered power (integrated in $freq$).
  
  $$P_{tot} \propto \left( \frac{\delta n_e}{n_e} \right)^2$$

- $\left(\frac{R}{L_{Te}^{exp}}\right) - \left(\frac{R}{L_{Te}^{crit}}\right)$ determines linear threshold for instability.

  - $t < 320 \text{ ms}$ \(\left(\frac{R}{L_{Te}^{exp}}\right) \sim \left(\frac{R}{L_{Te}^{crit}}\right)\)
    - ETG marginally stable, no fluctuations.

  - $t > 320 \text{ ms}$ \(\left(\frac{R}{L_{Te}^{exp}}\right) > \left(\frac{R}{L_{Te}^{crit}}\right)\)
    - Fluctuations develop.

  - $360 \text{ ms} < t < \sim 520 \text{ ms}$ (gray shading)
    - Similar $\left(\frac{R}{L_{Te}^{exp}}\right) - \left(\frac{R}{L_{Te}^{crit}}\right)$ produces VERY different $P_{tot}$. Nonlinear evolution of turbulence motivates the use of nonlinear gyro-kinetic simulations (future work).
Time Traces of Local Electron Density Gradient Confirm its Influence on Observed Fluctuation Amplitude

- As $R/L_{ne}$ increases, it dominates in Jenko’s formula $(R/L_{Te})_{crit} (t < 340 \text{ ms}, t > 410 \text{ ms} \& t > 515 \text{ ms})$. Fluctuations decrease during that time.

- Previous to $t \sim 320 \text{ ms}$ ETG is marginally stable with respect to Jenko critical gradient. No fluctuations are observed.

- $R/L_{ne}$ has a stabilizing effect when it dominates Jenko critical gradient.

• Lower-\(k\) \((k_{\perp} \rho_s < 10)\) \((\delta n_e / n_e)^2\) decreases for \(398 < t < 498\) ms.
• After \(t \sim 448\) ms, higher \(k\) \((k_{\perp} \rho_s \sim 12-16)\) fluctuation levels increase. During that time, \(R/L_{ne}\) increases.
Critical Gradient Computed with GS2 Linear Runs Agrees with Jenko’s Critical Gradient

- Regime of validity of \((R/L_{Te})_{crit}\):
  - low-\(\beta\)
  - positive \(s > 0.2\)
  - not NSTX parameters. large aspect ratio

- \((R/L_{Te})_{crit}\) is explicitly calculated with GS2.

- Good agreement between GS2 \((R/L_{Te})_{crit}\) calculations and Jenko \((R/L_{Te})_{crit}\).

  \(\rightarrow\) Jenko’s critical ETG formula is assumed valid in these NSTX plasmas.

- GS2 \((R/L_{Te})_{crit}\) seems to follow \(R/L_{ne}\).
GS2 Linear Simulations Show the Wavenumbers at Maximum Growth Rate Shift to Higher $k$ in Time

- Linear simulations compute most unstable mode ($k_r = 0$). Experimental $k$ is found to be linearly stable.
- Low-$k$ linear growth rates ($k_b \rho_s \leq 1$) are comparable to ExB shearing rate levels (Waltz, Miller PoP 1999).
- High-$k$ wavenumbers corresponding to maximum linear growth rate shift towards higher-$k$.
- Observed fluctuations decrease as $k_b \rho_s (\gamma_{\text{max}})$ increases.

### GS2 Simulations

<table>
<thead>
<tr>
<th>Channel</th>
<th>Shot</th>
<th>$k_b \rho_s$</th>
<th>$f$ (kHz)</th>
<th>$t$ (s)</th>
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</thead>
<tbody>
<tr>
<td>1</td>
<td>141767</td>
<td>$\sim 13-17$</td>
<td>$-3$ to $3$</td>
<td>0.398</td>
</tr>
</tbody>
</table>

### Experimental Data

- $f$ (MHz) vs. $t$ (s)
- $P$ (dB) vs. $f$ (MHz)
- $\gamma / C_s$ vs. $k_b \rho_s$

### Simulated Data

- $k_{\perp \text{sim}} = k_b$ ($k_r = 0$)
- $\gamma / C_s$
- Shift to higher $k$

### Graphs

- Linear growth rates (GS2) and WM ExB shearing rate (TRANSP A02)
- $t = 0.398, 0.448, 0.498, 0.565$ s
Wavenumber at Maximum Linear Growth Rate Correlates to Electron Density Gradient and Fluctuation Amplitude

- Linear growth rates are calculated at each time: determine $\gamma_{\text{max}}/(c_s/a)$ and $k_b\rho_s(\gamma_{\text{max}})$ (black dot).

- $\gamma_{\text{max}}/(c_s/a)$ not correlated with and $P_{\text{tot}}$ or $R/L_{ne}$.

- $k_b\rho_s(\gamma_{\text{max}})$ correlates to total scattered power $P_{\text{tot}}$ and $R/L_{ne}$ at the scattering location (cf. evolution within time panels).
Scan with GS2 is Performed to Confirm Effect of Electron Density Gradient on High-k Turbulence

- Real frequency $\omega_r$ and linear growth rate $\gamma$ are sensitive when $0.8 * R/L_{ne}$ dominates Jenko’s critical gradient for ETG.

- $t = 398$ ms, $0.8 * R/L_{ne}$ term not dominant $\Rightarrow$ $\gamma$ insensitive to $R/L_{ne}$.

- When $R/L_{ne}$ dominates, $R/L_{ne}$ decreases $\gamma$ and shifts $k_b \rho_s (\gamma_{max})$ to higher-k (cf. $t = 565$ ms) $\Rightarrow$ stabilizing effect.

- $|\omega_r|$ decreases with $R/L_{ne}$.
Experimental Real Frequency of High-k Turbulence is Calculated by Subtracting Doppler Shifted Frequency

- Lab frame frequencies detected $f_{lab}$ are Doppler shifted from plasma frame frequencies by $f_D = k_t v_t / 2\pi$, and $\omega_p / 2\pi = f_{lab} - f_D$.
- Obtain $k_t$ from ray tracing calculations, $v_t$ from CHERS measurement and TRANSPI calculations.

![Graph showing experimental real frequency of high-k turbulence calculation](image)

- $P_{scat}$
- $f_D \approx k_t v_t / 2\pi$
- $f_{lab}$
- $t = 398$ ms

- $\omega_p / (c_s / a)$
- $k \rho_s^{exp}$
- $t = 0.398$ s
- $t = 0.448$ s
- $t = 0.498$ s
- $t = 0.565$ s

<table>
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<tr>
<th>Channel 1</th>
<th>$f$ [MHz]</th>
<th>$P$ (dB)</th>
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Experimental Real Frequency and GS2 Real Frequency Exhibit Similar Behavior

**Exp:** $k_{\perp}^{exp} = \sqrt{(k_r^2 + k_b^2)}$, $k_r/k_b >> 1$

**Sim:** $k_{\perp}^{sim} = k_b$ ($k_r = 0$) (Most unstable mode)

- Experimental $k$ is linearly stable in GS2.
- $|\omega_p^{exp}|$ and $|\omega_r^{sim}|$ increase in time.
- Only 15% change in $c_s/a$ (normalization).
- Note $R/L_{ne}$ increases in time.
Correlation Between GS2 Wavenumbers at Maximum Growth Rate, Real Frequency and Electron Density Gradient

Compute $\gamma_{\text{max}}$, $k_b \rho_s(\gamma_{\text{max}})$ and $\omega_r/(c_s/a)$ @ ($k_b \rho_s=30$) in time, compare to $R/L_{ne}$.

- Low correlation between $\gamma_{\text{max}}$, and experimental $R/L_{ne}$.
- Correlation between $k_b \rho_s(\gamma_{\text{max}})$, $\omega_r/(c_s/a)$ @ ($k_b \rho_s=30$) and $R/L_{ne}$. 

$\gamma_{\text{max}}$, $k_b \rho_s(\gamma_{\text{max}})$ and $\omega_r/(c_s/a)$ plots are shown with regression lines and $R^2$ values.
Correlation Between Experimental Wavenumber at Maximum Fluctuation Amplitude and Density Gradient

Compute \((\delta n_e/n_e)^2_{\text{max}}, k_{\perp} \rho_s @ (\delta n_e/n_e)^2_{\text{max}}, \text{ and } \omega_p/(c_s/a) @ (k_b \rho_s = 13.2)\) and compare to \(R/L_{ne}\).

- Low correlation between \((\delta n_e/n_e)^2_{\text{max}}, \omega_p/(c_s/a)\) and experimental \(R/L_{ne}\), but note similar trend as found from GS2 linear simulations (slide 20).
- Correlation between \(k_{\perp} \rho_s @ (\delta n_e/n_e)^2_{\text{max}}\) and \(R/L_{ne}\) (possible beam refraction effects on \(k_{\perp}\)).
Summary

• High-k **electron scale density fluctuations** are detected with the coherent microwave scattering diagnostic at NSTX.
• \((R/L_{Te}^{\text{exp}}) - (R/L_{Te})_{\text{crit}}\) determines linear threshold for instability, and correlates to presence of observed fluctuations.
• As local **electron density gradient** \((R/L_{ne})\) increases, it dominates Jenko’s critical ETG and is observed to have a stabilizing influence on observed fluctuations.
• Increasing \(R/L_{ne}\) produces a shift of high-k fluctuations to even higher \(k\) values (stabilizing) and decreases real frequency \(\omega_r\).
• A scan on local \(R/L_{ne}\) with GS2 linear runs shows linear growth rate \(\gamma\) and real frequency \(\omega_r\) can be very sensitive to local \(R/L_{ne}\) when it is the dominant term in Jenko’s **critical ETG**.

Future Work

• Carry out further studies for other NSTX shots with similar characteristics and compare the influence of local electron density gradient.
• Perform transport analysis to study influence of local electron density gradient in electron thermal transport.
• Carry out nonlinear gyrokinetic simulations to evaluate the effects of electron density gradient on turbulence and electron thermal transport.
Back-up slides
High-k Fluctuations Start after Small Spike in $D_\alpha$ and Mirnov Signal

- Before $t \sim 290$ ms, MHD activity is high. At ~290 ms, an ELM event takes place and MHD activity quiets.

- Between $t \sim 290$ ms and $t \sim 320$ ms, high-k fluctuations are absent and MHD activity is quiet.

- **High-k fluctuations** start at $t \sim 320$ ms, after small ELM event, detected in $D_\alpha$ and Mirnov signal.
Typical quantities in these NSTX D plasmas

- Measured fluctuation wavenumbers $k_{\text{perp}} \sim 20 \text{ cm}^{-1} \sim 2000 \text{ m}^{-1}$
- $\omega_{\text{pe}} = 2\pi*90\text{GHz}*\sqrt{\text{ne}(10^{20}\text{[m}^{-3}]}) \sim 3.6*10^{11} \text{ s}^{-1}$
- $f_{\text{pe}} \sim 57 \text{ GHz}$
- $\omega_{\text{pD}} = \omega_{\text{pe}}/\sqrt{\text{mi/me}} \sim 5.9*10^{9} \text{ s}^{-1}$
- $f_{\text{pD}} \sim 0.94 \text{ GHz}$
- $\Omega_{\text{ce}} = 2\pi*(28\text{GHz}/\text{Tesla}) \sim 8.8*10^{10} \text{ s}^{-1}$
- $f_{\text{ce}} \sim 14 \text{ GHz}$
- $\omega_{\text{pe}}/\omega_{\text{ce}} \sim 4 >> 1$ (no ECH)
- $\omega_{\text{cD}} = 2\pi*(7.6\text{MHz}/\text{Tesla}) \sim 2.4*10^{7} \text{ s}^{-1}$
- $f_{\text{cD}} \sim 3.8 \text{ MHz} >> \text{drift wave fluct (low-f)}$
- $V_{\text{te}} = \sqrt{2}*4.2*10^{5}\text{[m/s]}*\sqrt{\text{Te}[\text{eV}]} \sim 1.3*10^{7} \text{ m/s}$
- $c_s = \sqrt{2}\sqrt{\text{mi/me}} \sim 3.03*10^{5} \text{ m/s}$
- Debye length $\lambda_{\text{de}} = v_{\text{te}}/(\sqrt{2}\omega_{\text{pe}}) \sim 2.6*10^{-5} \text{ m}$
- electric collisionless skin depth $\delta_e = c/\omega_{\text{pe}} \sim 8.8*10^{-4} \text{ m}$
- Alfven velocity $v_{\text{A}} = c*(f_{\text{ci}}/f_{\text{pi}})/\sqrt{1+(f_{\text{ci}}/f_{\text{pi}})^2} \sim c*f_{\text{ci}}/f_{\text{pi}} \sim 1.21*10^{6} \text{ m/s}$
- Tor. Rotation vel. $v_{\text{t}}$ (CHERS) $\sim 70 \text{ km/s}$
- $\beta = c_s^2/v_A^2 \sim 0.06$
- $\rho_{\text{e}} = v_{\text{te}}/(\sqrt{2}\omega_{\text{ce}}) \sim 0.1 \text{ mm}$
- $\rho_{\text{s}} = c_s/(\sqrt{2}\omega_{\text{ci}}) \sim \rho_{\text{e}}*\sqrt{\text{mi/me}} \sim 0.6-0.7 \text{ cm}$
Spatial Localization and Wavenumber Resolution

- Volume overlap of incident and scattered beams leads to poor spatial localization.
- Theory [cf. Horton Rev. Mod. Phys. 1999] predicts $k_i \sim 1/qR \ll k_\perp \Rightarrow \vec{k} \cdot \vec{B} \approx 0$

- Plasma fluctuations must satisfy:
  \[
  \begin{align*}
  k \cdot B & \approx 0 \quad (1) \quad \text{Perpendicular fluctuations.} \\
  k & = 2k_i \sin(\theta_s / 2) \quad (2) \quad \text{Bragg Condition}
  \end{align*}
  \]

- When incident beam forms a small angle with $\mathbf{B}$, (1) and (2) become highly dependent on \textit{toroidal curvature} of magnetic field (cf. scattered beams at $P_1$ and $P_2$ in the figure). \textbf{Oblique propagation} (outside the midplane) of incident beam exploits this phenomenon and enhances \textit{longitudinal localization} of fluctuations [cf. Mazucatto Phys. Plasmas 2003].

- For \textit{midplane propagation}, (1) and (2) are only satisfied at $P_1$ and $P_2$ and fluctuation wavenumber is purely in the \textbf{radial direction}.

- In practice, beam propagation is out of midplane, but oblique angle is small ($\sim 5^\circ$). $\mathbf{k}$ is \textit{mostly} radial.

- Gaussian beam width dictates $k$ and $R$-resolution
  \[
  A(r_\perp) = \exp(-r_\perp^2 / \omega_0^2) \\
  G(k_\perp) = \exp(-k_\perp^2 / \Delta k^2) \\
  \Delta k = 2 / \omega_0
  \]
Collective Thomson Scattering Theory is used to measure ETG-scale turbulence

- Collective/coherent and incoherent scattering

  \[ \lambda_D \]
  
  \[ e^- \]
  
  \[ k \lambda_D \leq 1 \]

  \[ \lambda_D \]
  
  \[ e^- \]
  
  \[ k \lambda_D \geq 1 \]

- Typical values (NSTX)  \( \lambda_D \sim 10^{-5} \text{ m}, k \sim k_{\perp} < 10^4 \text{ m}^{-1} \) (high-\(k\))

  \( k \lambda_D < 1 \) (collective scattering)

- Scattered power density

  \[
  \frac{d^2 P}{d\Omega dv} = P r_e^2 L_z |\Pi \cdot \hat{e}|^2 \left| \tilde{n}_e(k, \omega) \right|^2 \frac{1}{VT}
  \]

  - \( r_e \) classical electron radius
  - \( V, L_z \) volume and length of scattering volume
  - \( \Pi \) polarization tensor
  - \( \hat{e} \) direction of incident electric field
  - \( T \) observation time
A Scan on $R/L_{Te}$ is Performed to Compute a Critical Gradient with GS2 Linear Runs

- $R/L_{Te}$ is varied keeping all other quantities constant. The factor is called $(R/L_{Te \text{ fac}})$.
- High-$k$ linear growth rates saturate with decreasing $(R/L_e)$.
- $(R/L_{Te \text{ fac}})_{\text{crit}}$ is found to be the minimum $R/L_{Te}$ to satisfy $\gamma = 0$. 

![Graph showing linear relationship between $\gamma/(C_s/a)$ and $k_\theta \rho_s$.](image)