Nonthermal Particle and Full-wave Effects on Heating and Current Drive in the ICRF and LHRF Regimes

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Summary

• Improved specifications of RadioFrequency (RF) simulation models allow higher fidelity to experiments than ever before.
  – Non-Maxwellian dielectrics allow incorporation the effects of alpha particle populations in fusion plasmas, RF induced energetic tails, and neutral beams on wave propagation and damping.
  – Coupling of RF and Fokker-Planck codes within the framework of the new non-Maxwellian dielectric response permits self-consistent, weakly-nonlinear simulations of RF heated plasmas.
  – Ultra high resolution make the lower hybrid (LH) frequency range accessible to full wave simulations at experimentally relevant parameters.

• These advances lead to new insights in RF physics.
  – The role of fusion generated alphas on power deposition from RF.
  – Full wave diffraction as a possible mechanism for filling in the “spectral gap” in LH propagation, i.e., explaining the rapid slowing down of the phase velocity of LH waves to a few times the electron thermal speed.
Integrated RF modeling requires a number of interconnected components

3D Maxwell solver with simplified plasma boundary conditions

Wave equation solver

Fokker Planck equation

Antenna

Launched spectrum

Wave propagation/absorption

wave fields

Plasma response

Antenna/edge interactions

Stand alone models

Plasma dynamics (transport, stability) $T_j(r), n_j(r), j(r), v_j(r), B_0$

Integration

Experimental data

Integrated transport code.
Basic equations of wave propagation and absorption

\[ \nabla \times \nabla \times \mathbf{E} + \frac{\omega^2}{c^2} \mathbf{E} = \mathbf{J}_p \circ \mathbf{E} + \mathbf{J}_{\text{ant}} \]  

\[ \text{plasma wave current: an integral operator on } \mathbf{E} \]

- Time harmonic \( \leftrightarrow \) real \( \omega \), coherent waves, spatial damping
- \( \mathbf{J}_{\text{ant}} = \) antenna source current from a variety of antenna models
- Boundary conditions: bounded domain – conducting or inhomogeneous source region
- Weakly non-linear, time average distribution function \( f_0(v, t) \) evolves slowly:

\[ f(x, v, t) = f_0(x, v, t) + f_1(x, v) e^{-i\omega t} \]

slow, quasilinear time scale \( \sim \tau_E \) 

- Fast, RF time scale

\[ \mathbf{J}_p = \text{fluctuating plasma current due to wave} \rightarrow \text{non-local, integral operator on } \mathbf{E} \]

\[ J_p(x, t) = e \int d^3 v \ v f_1(x, v, t) \quad f_1(x, v, t) = -\frac{e}{m} \int_{-\infty}^{t} dt \mathbf{E}_1(x', v', t', t') \cdot \frac{\partial f_0}{\partial v} \]

- Approximate operator locally by integrating along guiding center orbits

- Effectively uniform plasma conductivity (Stix) \( \rightarrow \) \( \sigma(k_\parallel, k_\perp, \omega) \Rightarrow I_\perp(k_\perp \rho) \cdot Z \left( \frac{\omega - \gamma \Omega_c}{k_\parallel v_{th}} \right) \)
Advances made in the US SciDac Initiative in two full-wave ICRF solvers are shown

- All Orders Spectral Algorithm (AORSA) – 1D, 2D & 3D (Jaeger)
  - Spectral in all 3 dimensions
  - Cartesian/toroidal coordinates
  - Includes all cyclotron harmonics
  - No approximation of small particle gyro radius $\rho$ compared to wavelength $\lambda$
  - Produces huge, dense, non-symmetric, indefinite, complex matrices
TORIC FullWave Code Description

- TORIC – 2D (Brambilla/Bonoli/Wright)
  - Mixed spectral (toroidal, poloidal), finite element (radial)

\[ \nabla \times \nabla \times E = \frac{\omega^2}{c^2} \left\{ E + \frac{4\pi i}{\omega} (J^P + J^A) \right\} \leftarrow E(x) = \sum_m E_m(\psi) \exp(imb + in\phi) \]

  - Flux coordinates
  - Up 2^{nd} cyclotron harmonic
  - Expanded to 2^{nd} order in \rho/\lambda
  - Sparse banded matrices

- Where J^A and J^P are the antenna and plasma currents.

- The TORIC dielectric models include IBW, ICW, and FW dispersion, and fast and slow lower hybrid waves in a separate module with a current strap antenna model for FW launch and a waveguide model for LH launch.

- Resolution requirements depend on the specific wave scenario being modeled.
  - ModeConversion requires \sim 255 poloidal modes
  - LH slow waves require \sim 1000 poloidal modes
What advances were needed?

- Adequate description of parallel plasma response – beyond the Z function, generalized Z function of Smithe and Colestock

- Higher harmonics – $\omega > 2 \Omega_{ci}$ – need all orders dielectric

- Non-Maxwellian equilibrium distributions – general distribution in RF conductivity for wave codes, coupling to Fokker-Planck solvers for self consistency

- Computer power/code acceleration
  - Much larger memory and/or out of core techniques to allow higher resolution
  - Much higher processing speed/massive parallelization to allow improved physics and higher resolution
  - Algorithm optimization
A conductivity module allowing general plasma distributions has been developed – all orders, all cyclotron harmonics, shared by all wave solvers

Conductivity tensor

$$\tilde{\mathbf{K}}(x, k_x) = \mathbf{1} + \left( 1 - i \frac{\partial}{\partial k_{2,x}} \cdot \frac{\partial}{\partial x} \right) \tilde{\mathbf{W}}(x, k_{1,x}, k_{2,x}) \bigg|_{k_{1,x} = k_{2,x} = k_x}$$

$$\tilde{\mathbf{W}}(x, k_1, k_2) = \frac{\omega_p^2}{\omega^2} \sum_{n=-\infty}^{\infty} \exp(in(\beta_1 - \beta_2)) \mathbf{C}^{-1}(\beta_1) \cdot \tilde{\Theta}_n \cdot \mathbf{C}(\beta_2)$$

**Rotation matrices (Lab frame - local magnetic frame)**

Kernel in Stix frame

$$\tilde{\Theta}(x, k_1, k_2) \equiv \int_0^{\infty} d\tau \int_0^{\infty} dv_\parallel e^{i(\omega \tau - n\theta(\tau) - k_\parallel, v_\parallel \tau)} \int_{-\infty}^{\infty} dv_\perp \tilde{\mathbf{w}}(x, v, k_1, k_2)$$

**General expression for \( \mathbf{w} \) (arbitrary distribution)**

$$\mathbf{w} \equiv \begin{bmatrix} \frac{v_\perp}{\sqrt{2}} J_{n+1}(\xi_2) \\ \frac{v_\perp}{\sqrt{2}} J_{n-1}(\xi_2) \\ v_\parallel J_n(\xi_2) \end{bmatrix} \begin{bmatrix} \hat{L} f_0 J_{n+1}(\xi_1) \\ \frac{\hat{L} f_0}{\sqrt{2}} J_{n-1}(\xi_1) \\ \hat{L}_n f_0 J_n(\xi_1) \end{bmatrix}$$

**Differential operators**

$$\begin{cases} \hat{L} f_0 & \equiv \left( 1 - \frac{k_\parallel v_\parallel}{\omega} \right) \frac{\partial f_0}{\partial v_\perp} + \frac{k_\parallel v_\perp}{\omega} \frac{\partial f_0}{\partial v_\parallel} \\ \hat{L}_n f_0 & \equiv \frac{n\Omega_0}{\omega} \frac{\partial f_0}{\partial v_\parallel} + \left( 1 - \frac{n\Omega_0}{\omega} \right) \frac{\partial f_0}{\partial v_\parallel} \end{cases}$$

**Tremendous increase of computational requirements**
Self-consistency: past dielectric models assumed that $f_0$ is Maxwellian

- High power waves can drive the distribution far from Maxwellian
- Significant non-Maxwellian components can be produced by neutral injection or fusion alpha particles

Non-Maxwellian distributions can:
- Affect local damping rate and wavelength
- Modify heating and current drive profiles
- Change partition of power deposition among plasma species
- Affect mode-conversion

Calculation of wave fields consistent with the plasma distribution requires closed loop coupling of four significant physics models
CQL3D: Bounce-averaged Fokker-Planck Code is used to generate non-Maxwellian distributions

Solve for bounce-averaged distribution at torus equatorial plane ($\theta_p = 0$), $f_0(\rho, v_{||}, v_{\perp}, t)$

$$\frac{\partial (\lambda f)}{\partial t} = \frac{\partial}{\partial \nu} \left[ G_E + G_{RF} + G_{coll} \right] + R(f) + S_{NB} + S_{NB} + L$$

$\rho =$ generalized radial coordinate labeling (non-circular) flux surface

$\lambda =$ field line connection length

$\Gamma_E =$ velocity space flux due to toroidal electric field (Ohmic)

$\Gamma_{RF} = D_{QL} \frac{\partial f}{\partial \nu} =$ velocity space flux due to full, bounce average, RF Quasi-linear operator (all harmonics, Bessel functions, all wave modes)

$\Gamma_{coll} =$ full, nonlinear, 2D, relativistic collisional operator

$R(f) =$ Radial diffusion and pinch operator with $v$ dependent coefficients

$S_{NB} =$ Monte Carlo neutral beam source (NFREYA)

$S_{KO} =$ Knock-on collisions (for electrons)

$L(v) =$ velocity dependent prompt loss term

- Implicit solve in $\rho, v_{||}, v_{\perp}$. Operator splitting in 3D cases.

- 2D full wave RF fields are being imported in a “local spectral representation”
We see significant differences compared to two temperature Maxwellian model:
- Much narrower ion absorption zones around high harmonic cyclotron resonances
- Much less deposition into electrons: 15%/41%, more power into D: 81%/52%

Non-Maxwellian presently requires about 13$\times$ more CPU time – but have already sped the algorithm by 25$\times$ and we expect to close the gap
First converged 2D full wave solutions for ITER scale device, with non-Maxwellian distribution

Calculations confirm that fast alpha absorption can be held to low levels with appropriate choice of frequency

Fast alpha population does effect anti-Hermitian part of conductivity enough to modify wave propagation
Status of LHRF Wave Propagation Codes

- Ray tracing in 2D and 3D still the preferred method
  - ACCOME, LSC, FRTC, CURRAY, etc
- Full-wave studies are now elucidating important physics effects (focusing and diffraction) not easily included in geometrical optics treatments [e.g., Pereverzev, NF (1992)].
- **Full-wave studies possible through major rewrite of TORIC solver:**
  - Formulation of the RF conductivity operator in the LHRF
    - Unmagnetized ions, strongly magnetized electrons
    - 4th order system, two waves – fast and slow LH
  - New boundary conditions at plasma wall to impress $E_{\parallel}$
    - FW BCs had current strap model to impose $E_{\perp}$
    - LH BCs use waveguide model:
      $$E_{[n,\xi]}^{(m)} = -e^{-i\theta_\xi}e^{-im\Delta_\varphi} + \frac{1}{4m^2 \Delta^2_\varphi - \pi^2} \sin \Theta_w, \cos \Theta_w$$
  - Massive parallelization of field solver achieves converged solutions with 1000 radial elements and 1023 poloidal modes.
LHRF Physics Regime—cold ions and magnetized electrons

- Frequency range is intermediate of ion and electron cyclotron frequencies:

\[ \Omega_{ci} \ll \omega \ll \Omega_{ce} \]

\[ k_{\perp}^2 \approx -\frac{\epsilon_{\parallel}}{\epsilon_{\perp}} k_{\parallel}^2 \]

\[ \epsilon_{\perp} = 1 + \frac{\omega_{pe}^2}{\Omega_{ce}^2} - \frac{\omega_{pi}^2}{\omega^2} \approx 1 \]

\[ \epsilon_{\parallel} = 1 - \frac{\omega_{pe}^2}{\omega^2} - \frac{\omega_{pi}^2}{\omega} \approx -\frac{\omega_{pe}^2}{\omega} \]

\[ \rightarrow k_{\perp} \approx \frac{\omega_{pe}}{\omega} k_{\parallel} \]

- Fourth order DR (with fast and slow LH waves) predicts an accessibility criteria

\[ n_{\parallel} > n_a = \frac{\omega_{pe0}}{\Omega_{ce0}} + \epsilon_{\text{perp}0}^{1/2} \]

- Above accessibility, no ion-plasma wave.

Typical Parameters:

- \[ B_0 = 4.5 \; T \], \[ D^+ \]
- \[ f_0 = 4.6 \; GHz \]
- \[ n_{\parallel} = 2.5 \], \[ n_a = 2 \]
- \[ n_{e0} = 2 \times 10^{20} \; m^{-3} \]
- \[ \omega / \Omega_{cD} \approx 125 \]
- \[ k_{\perp} \approx 66 \; cm^{-1} \]
At 4.6 GHz, a FW polarization is launched from the low field side. ($n_\parallel = 1.5$, $n_{acc} = 2$)

- It is propagative at $X = 19 \text{ cm}$, and undergoes mode transformation to slow LH wave at $X = 15 \text{ cm}$

- Loci of LH reflections can be seen in the full wave field patterns - forming caustics.

**Plasma Parameters:**
- $D(5\% \text{ H})$, $n_\parallel = 1.5$, $f = 4.6 \text{ GHz}$
- $B_0 = 5.3 \text{ T}$, $T_e = 3.5 \text{ keV}$, $T_i = 2.0 \text{ keV}$
- $n_e(0) = 1.5 \times 10^{20} \text{ m}^{-3}$, $I = 1 \text{ MA}$
- $N_m = 1023$, $N_r = 960$
Cold Plasma DR slow(*) and fast

The FW is coupled at the edge, then mode converts to a slow wave at the edge cutoff and reflects inward again to the confluence. All wave power is absorbed on the electrons by Landau damping.

There is a suggestion of resonance cones in the full wave field patterns. DR describes "Ring" of LH waves and there is a Caustic formation is also apparent. Caustic formation is also apparent. Caustic formation is also apparent. Caustic formation is also apparent.

Waves describe "Ring" of LH
Spectral shift is large at caustic

- The distribution of $n_\parallel$ on flux surfaces shows a significant upshift from an averaged launched $n_\parallel$ of 2 to $>4$ in the middle of the annulus at $r/a=0.75$.

- Note, that the spectral broadening right at the antenna from geometric effects is comparable to that seen in raytracing.

- This rapid upshift at the caustic causes all the power to be absorbed in the narrow region bound by the caustic and the edge cutoff - an effect not predicted in geometrical optics!
Ray tracing simulations show important differences

- The local $n_\parallel$ evolves to 2.5 on the high field side from 1.5 at the antenna.
- $N_\parallel$ evolution is due primarily to geometric effects of major radius position, and results in different damping.
- Rays penetrate farther into the plasma and damp, whereas damping in the full-wave simulation is confined to an annulus.
Power deposition profiles are both localized, but at slightly different radii

- TORIC shows power deposition is confined to a narrow region of $r/a=[0.65,0.83]$
- ACCOME has damping penetrating to the center and starting within the 'caustic'.
- The difference may be related to the downshifting of $v_\phi$ by diffraction, which is high at the caustic - evidence for full wave enhancement in $k_\parallel$ filling the spectral gap.
Conclusions

- Using the new non-Maxwellian dielectric model in AORSA-2D with particle distributions calculated by CQL-3D we see much narrower ion absorption zones around high harmonic cyclotron resonances and much less deposition into electrons – it is important to get the number of fast particles in the distribution correct, the bi-Maxwellian model overestimates this.

- We have evaluated the parasitic effects of alpha particle damping for ICRF heating in the proposed ITER device using a slowing down distribution and found the effect to be negligible (less than 5% of the total ICRF power) and confirmed parasitic absorption may be avoided using appropriate wave frequencies.

- Full-wave phenomena in the LHRF regime can now be studied for the first time ever in toroidal geometry at experimentally relevant parameters, allows projections of LH on ITER, and upcoming Alcator C-Mod experiments.

- Diffraction effects may provide a well-defined mechanism by which LH waves are able to damp efficiently at \((2-3)v_{te}\), despite having been injected at phase speeds of \((5-10)v_{te}\), thus providing a simple explanation of the lower hybrid spectral gap problem.