Developing simple physical descriptions of stagnation in the presence of non-radial flows

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Making connections between 3D simulation and 1D theory is beneficial.
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3D (Z pinch)

1D theory can give an underlying picture of the stagnation dynamics.
Making connections between 3D simulation and 1D theory is beneficial. 3D simulation can inform how 1D theory/simulations should be modified to account for 3D effects. 1D theory can give an underlying picture of the stagnation dynamics.
Making connections between 3D simulation and 1D theory is beneficial.

3D (Z pinch)

Questions:
1. What 1D physics is important in driving residual flow at stagnation?
2. Effects of residual flow on pressure/energy balance at stagnation?
3. Can a 1D model approximately describe 3D stagnation?
Z pinches use implosion/stagnation to achieve high energy density (HED)

Spherical: ICF capsule

\[ \text{DT gas} \quad R \sim 1 \text{ mm} \]

\[ \text{Ablator} \]

\[ \text{DT fuel} \]

Implosion\[ v \sim 3 \times 10^5 \text{ m/s} \]

Stagnation

DT fuel

Cylindrical: MagLIF (ICF)

\[ \text{Metallic liner} \]

\[ \text{B}_z \]

\[ j \times B \]

\[ B_\theta \]

Cylindrical: wire array Z pinch (radiation source)

\[ R \sim 10 \text{ mm} \]

\[ L \sim 10 \text{ mm} \]

These arrays constitute hundreds of tiny wires \( (R_w \sim 5 \mu\text{m}) \), although I am showing only 4.
Z pinches use implosion/stagnation to achieve high energy density (HED)

Spherical: ICF capsule

- DT gas
- Ablator
- DT fuel
- R ~ 1 mm
- X-ray/laser radiation

Implosion:
- \( v \approx 3 \times 10^5 \) m/s

Stagnation:
- DT fuel

Cylindrical: MagLIF (ICF)

- Metallic liner
- \( B_z \)
- \( B_\theta \)
- \( j \times B \)

Cylindrical: wire array Z pinch (radiation source)

- These arrays constitute hundreds of tiny wires
  \( (R_w \approx 5 \mu m) \), although I am showing only 4.
Z pinches use implosion/stagnation to achieve high energy density (HED)

Spherical: ICF capsule
- DT gas
- R~1 mm
- X-ray/laser radiation
- DT fuel
- Ablator

implosion
- $v \sim 3 \times 10^5$ m/s

stagnation
- DT fuel

Cylindrical: MagLIF (ICF)
- Metallic liner
- $B_z$
- $j \times B$
- $B_\theta$

Cylindrical: wire array Z pinch (radiation source)
- $B_z$
- $j \times B$
- $j_z$
- $B_\theta$

This is a gross oversimplification

Like ICF capsules/MagLIF, the Z pinch is 3D

Spherical: ICF capsule

Cylindrical: MagLIF (ICF)

Cylindrical: wire array Z pinch (radiation source)

3D Simulation

D.S. Clark et al., PoP 22, 022703 (2015)

J.R. Rygg et al., PRL 112, 195001 (2014)

T.J. Awe et al., PRL 111, 235005 (2013)


C. Jennings

X-ray radiography

Time-integrated self-emission

(FWHM~50-100 µm)

D.B. Sinars et al., PRL 100, 145002 (2008)

X-ray radiography

Time-integrated self-emission

(peak x-ray power)

Like ICF capsules/MagLIF, the Z pinch is 3D

**Spherical: ICF capsule**
- DT gas: R~1 mm
- DT fuel
- Ablator

**MagLIF**
- Implosion
- Stagnation: R~200 μm

**Cylindrical: MagLIF (ICF)**
- Metallic liner
- Time-integrated self-emission: R~50 μm

**Cylindrical: wire array Z pinch (radiation source)**
- X-ray radiography: 1 mm
- X-ray radiography: 2 mm

We will focus on the Z pinch, but the physical effects we see here may have applicability to ICF systems.

D.S. Clark et al., PoP 22, 022703 (2015)
D.B. Sinars et al., PRL 100, 145002 (2008)
J.R. Rygg et al., PRL 112, 195001 (2014)
3D wire-array Z pinch simulation

\( \rho = 1 \text{ kg/m}^3 \)

**Alegra:** 3D radiation MHD + thermal conduction

Eulerian mesh

Artificial viscosity

1.7 million elements (dz\(\sim\)60 \(\mu\)m, dr\(\sim\)20 \(\mu\)m)

Mass injection scheme*

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3D wire-array Z pinch simulation

Simulate 1.15 mg, tungsten compact array \((R_o=1\text{ cm})\)

At \(t=-2.4\ \text{ns}\), radiation is “turned off” \((\sigma/1e4)\)
3D wire-array Z pinch simulation

t=-2 ns

$\rho=1 \text{ kg/m}^3$

surface
What's the significance of this non-monotonic ram pressure?
1D shock solution

1D planar geometry

Consider cold fluid particles imploding towards a rigid boundary. Density and velocity profiles are flat.

Problem we will consider is purely hydrodynamic.

This is the “Noh problem”*

*W.F. Noh, J. Comp. Phys. 72, 78 (1987)
1D shock solution

Particle 1 collides into the boundary and converts all its kinetic energy into internal energy.
1D shock solution

Hot core grows outward through shock accretion.

\[ D = \frac{\gamma - 1}{2} v_0 \]

Rigid boundary

Hot core, zero residual flow
1D shock solution

\[ p_{ram} = \rho_0 v_0^2 \]

The hot core is confined in the sense that \( v = 0 \) there.

This confinement is due solely to the incoming ram pressure.
1D shock solution: increasing $p_{\text{ram}}$

The Noh problem has been generalized by A. Velikovich to allow for spatially varying $\rho(r)$, $v(r)$.

Density and ram pressure profiles are increasing.
1D shock solution: increasing $p_{\text{ram}}$

Because of increasing ram pressure, shocked fluid particle continues to compress.
1D shock solution: increasing \( p_{\text{ram}} \)

Because of continued compression in core, density there rises with time.

Residual flow in core
1D shock solution: decreasing $p_{\text{ram}}$

Cold fluid particles implode towards a rigid boundary. Density and ram pressure profiles are decreasing.
1D shock solution: decreasing $p_{\text{ram}}$

Because of decreasing ram pressure, shocked fluid particle expands outward into imploding plasma.

Residual flow in core
1D shock solution: non-monotonic $p_{\text{ram}}$

$p_{\text{ram}} = \rho v^2$

During rising portion of $p_{\text{ram}}$, core compresses

During falling portion of $p_{\text{ram}}$, core expands

3D simulation also shows non-monotonic $p_{\text{ram}}$
3D simulation fluid flows

$t = -1.6 \text{ ns}$

$z = 3.5 \text{ mm}$

$0.25 \text{ mm}$

$0.5 \text{ mm}$

$0.75 \text{ mm}$

$1 \text{ mm}$

$KE_{xy} \sim 4 \times KE_z$ during times of interest
Flow is not completely radial; plasma collides obliquely and off-axis.

$p = 10^{12}$ J/m$^3$

(partially stagnated plasma)

$z = 3.5$ mm

$1$ mm

$0.75$ mm

$0.5$ mm

$0.25$ mm

$DENSITY$ (kg/m$^3$)

$1.000e+03$

$1.000e+02$

$1.000e+01$

$1.000e+00$

$1.000e-01$

$z = 3.5$ mm

$1.000e+06$

$7.500e+05$

$5.000e+05$

$2.500e+05$

$0.000e+00$

$v$ (m/s)

$DENSITY$

$1.000e+03$

$1.000e+02$

$1.000e+01$

$1.000e+00$

$1.000e-01$

$DENSITY$

$z = 3.5$ mm

$DENSITY$

$z = 3.5$ mm

Flow is not completely radial; plasma collides obliquely and off-axis.
3D simulation fluid flows

$z=3.5$ mm

$0.5$ mm

$0.25$ mm

Shock
accretion

$\rho = 1 \times 10^{12}$ J/m$^3$

$t = -1.2$ ns

$DENSITY$ (kg/m$^3$)

$1.000e+03$

$1.000e+02$

$1.000e+01$

$1.000e+00$

$1.000e-01$

$v (m/s)$

$KE (MJ)$

$I (MA)/100$

$IE (MJ)$

$z = 3.5$ mm

$DENSITY$

$1.000e+03$

$1.000e+02$

$1.000e+01$

$1.000e+00$

$1.000e-01$

$DENSITY$

$1.000e+03$

$1.000e+02$

$1.000e+01$

$1.000e+00$

$1.000e-01$

$KE (MJ)$

$I (MA)/100$

$IE (MJ)$

$t (ns)$

$0.25 mm$

$0.5 mm$

Shock
accretion

$1.000e+06$

$7.500e+05$

$5.000e+05$

$2.500e+05$

$0.000e+00$

$1.000e+06$

$7.500e+05$

$5.000e+05$

$2.500e+05$

$0.000e+00$

$t (ns)$

$0.25 mm$

$0.5 mm$

Shock
accretion

$1.000e+06$

$7.500e+05$

$5.000e+05$

$2.500e+05$

$0.000e+00$

$t (ns)$

$0.25 mm$

$0.5 mm$

Shock
accretion

$1.000e+06$

$7.500e+05$

$5.000e+05$

$2.500e+05$

$0.000e+00$
Stagnated region (purple contour) is growing via accretion. “Fast” exit flow is forming.
3D simulation fluid flows

Exit flow is turned around by ram pressure, forming a vortex.
3D simulation fluid flows

Vortices generate centrifugal pressure

p = 1e12 J/m³

t = -0.4 ns

Z = 3.5 mm

EKIN (J)

KE (MJ)

I (MA)/100

IE (MJ)

t (ns)
Effect of residual flows on pressure balance

How much “pressure” do the vortices contribute?

\[ \rho \left( \frac{\partial \mathbf{v}}{\partial t} + \mathbf{v} \cdot \nabla \mathbf{v} \right) = -\nabla p + \mathbf{J} \times \mathbf{B} \]

radial component:

\[ \rho \frac{\partial v_r}{\partial t} = -\frac{\partial p}{\partial r} - \rho \left[ v_r \frac{\partial v_r}{\partial r} + \frac{v_\theta}{r} \frac{\partial v_r}{\partial \theta} + v_z \frac{\partial v_r}{\partial z} - \frac{v_\theta^2}{r} \right] \]

\[ + J_\theta B_z - J_z B_\theta \]

pressure gradient
centrifugal force
Centrifugal force is comparable to pressure gradient

How much “pressure” do the vortices contribute?

\[ \rho \left( \frac{\partial v}{\partial t} + v \cdot \nabla v \right) = -\nabla p + J \times B \]

radial component:

\[ \rho \frac{\partial v_r}{\partial t} = -\frac{\partial p}{\partial r} - \rho \left[ v_r \frac{\partial v_r}{\partial r} + \frac{v_\theta}{r} \frac{\partial v_r}{\partial \theta} + v_z \frac{\partial v_r}{\partial z} - \frac{v_\theta^2}{r} \right] \]

- pressure gradient
- centrifugal force

Y.M. Maron et al., PRL 111, 035001 (2013)
Are vortices generated in ICF capsules?


\[ \rho = \rho_0 \]

\[ v = -v_0 \]

radius (μm)

Density (g/cm³)

time (ns)

idealized fluid particle trajectories

Shock Wave

Ablation Front

Ablator

Fuel

\( t_1 \) (transmitted shock about to hit axis)
Are vortices generated in ICF capsules?

Possibly vortices are generated due to imbalance in pressure/ram pressure, as just described.
Are vortices generated in ICF capsules?

t_3 (shock reflection in gas complete)
Are vortices generated in ICF capsules?

Could vortices be compressed and amplified by transmitted shock?

However, viscosity is more important here than in a Z pinch:

\[
\nu_i (\text{cm}^2/\text{s}) = 3.3 \times 10^{-5} \sqrt{A[T(\text{eV})]^{5/2}} \frac{\sqrt{\Lambda Z^4 \rho (\text{g/cm}^3)}}{\ln \Lambda Z^4 \rho (\text{g/cm}^3)}
\]
How important is centrifugal force in ICF capsules?

Equation of motion:

$$\rho \left( \frac{\partial \mathbf{v}}{\partial t} + \mathbf{v} \cdot \nabla \mathbf{v} \right) = -\nabla p.$$ 

radial component (spherical geometry):

$$\rho \frac{\partial v_r}{\partial t} = -\frac{\partial p}{\partial r} - \rho \left[ v_r \frac{\partial v_r}{\partial r} + \frac{v_\theta}{r} \frac{\partial v_r}{\partial \theta} + \frac{v_\phi}{r \sin \theta} \frac{\partial v_r}{\partial \phi} - \frac{v_\theta^2 + v_\phi^2}{r} \right]$$

- pressure gradient
- centrifugal force
Core plasma expands outward into the imploding plasma due to decreasing $p_{\text{ram}}$. Total KE is still dropping.
Can a 1D shock solution describe 3D stagnation?

1D shock model has been useful in providing qualitative explanation.

How about quantitative comparison?

Use the generalized shock solution (A. Velikovich), allowing non-uniform initial $\rho(r)$, $v(r)$.

$\rho = \rho_0 \left( \frac{r}{R_0} \right)^{2\lambda}$

$\lambda > 0$

$v = -v_0 \left( \frac{r}{R_0} \right)^{-\lambda}$

$\lambda \neq -1$

Can a 1D shock solution describe 3D stagnation?

1D shock model has been useful in providing qualitative explanation.

How about quantitative comparison?

Use the generalized shock solution (A. Velikovich), allowing non-uniform initial $\rho(r), v(r)$.


\[
\rho_s(t) \propto t^{\frac{2(\chi - \lambda)}{1+\lambda}}
\]

\[
\rho_s(t) \propto t^{\frac{2\chi}{1+\lambda}}
\]

\[
R_s(t) \propto t^{\frac{1}{\lambda+1}}
\]
Pre-stagnation profiles exhibit “2 phase” profile

\[ \frac{\rho}{r^{1.6}} \]

\[ \rho \sim r^{0.5} \]

\[ \rho \sim r^{-1.6} \]

\[ p_s \sim t^{1.25} \]

\[ p_s \sim t^{4.25} \]

\[ R_s \sim t^{5/2} \]

\[ \rho_s \sim t^{-1.6} \]

\[ p_s \sim t^{-1.6} \]

\[ R_s \sim t \]
Comparison of shock solution with 3D simulation

\[ \langle \rho(r = 0) \rangle (\text{kg/m}^3) \]

\[ \langle p(r = 0) \rangle (\text{kg/m}^3) \]

\[ R(\text{mm}) \]
Shock solution agrees with 3D simulation during “phase 2”

Phase 1 describes initial accumulation of mass on axis. There isn’t sufficient symmetry for a 1D shock picture to be valid.

During phase 2, there is already a “large” core on axis, increasing the validity of a 1D shock model.

How might the residual kinetic energy during phase 1 dissipate?
Homogeneous (shockless) stagnation

$v(r)$ is linear. This allows particles to compress in unison, without shocks.

Finite $T$ is allowed. This results in a finite pressure gradient, causing particles to gradually slow down.
Stagnation is complete. All kinetic energy has converted to internal energy. The key parameter:

\[ \epsilon = \frac{\int p dV}{\int \frac{1}{2} \rho v^2 dV} \]

- Initial thermal energy
- Initial kinetic energy

Higher \( \epsilon \) (thermal energy) \( \Rightarrow \) less compression
Comparison of homogeneous stagnation with 3D simulation

Computing $\varepsilon$ from simulation, theory predicts too high a compression.

But suppose we artificially enhance $\varepsilon$, to account for the centrifugal pressure from the vortices.

$$\varepsilon = \frac{\int p dV}{\int \frac{1}{2} \rho v^2 dV}$$
Comparison of homogeneous stagnation with 3D simulation

\[ \varepsilon = \frac{\int p\,dV}{\int \frac{1}{2}\rho v^2\,dV} \]

Computing \( \varepsilon \) from simulation, theory predicts too high a compression.

But suppose we artificially enhance \( \varepsilon \), to account for the centrifugal pressure from the vortices.
Comparison of homogeneous stagnation with 3D simulation

Strong polar jets probably invalidate use of homogeneous stagnation solution in NIF capsule.

Questions:
1. What 1D physics is important in driving residual flow at stagnation?
   - ram pressure profile
2. Effects of residual flow on pressure/energy balance at stagnation?
   - centrifugal force due to vortices is comparable to pressure gradient
3. Can a 1D model approximately describe 3D stagnation?
   - Shock and shockless stagnation models are helpful in describing $\langle \rho(t) \rangle, \langle p(t) \rangle, R(t)$
Backup: wire array ablation

Spherical: inertial confinement fusion (ICF) capsule

DT gas

R~1 mm

X-ray radiation
(indirect drive)
Laser (indirect drive)

DT fuel

Ablator

Implosion

Stagnation

DT fuel

R~40 µm

Hot spot

Cylindrical: MagLIF (ICF)

Metall Linear

Bz

jxB

jz

Bθ

Bz

jxB

Bθ

Bθ

A hot, low density plasma corona forms around the wires, supporting current as well as jxB force

Cylindrical: wire array Z pinch (non-ICF)
Backup: wire array ablation

Spherical: inertial confinement fusion (ICF) capsule

- DT gas
- R~1 mm
- DT fuel
- X-ray radiation (indirect drive)
- Laser (indirect drive)

Cylindrical: MagLIF (ICF)

- Metallic liner
- \( B_z \)
- \( B_\theta \)
- \( j \times B \)

Cylindrical: wire array Z pinch (non-ICF)

- \( B_z \)
- \( B_\theta \)
- \( j \times B \)

During the ablation phase, outer edge of wires “cooks off” and converts to corona, which is then flung towards axis. Wires remain stationary.

X-ray radiation

stagnation

DT fuel

Hot spot (R~40 µm)
Backup: wire array ablation

Spherical: inertial confinement fusion (ICF) capsule

Stagnation

Hot spot (R~40 µm)

DT gas

R~1 mm

X-ray radiation (indirect drive)
Laser (indirect drive)

implosion

v~3e5 m/s

DT fuel

Cylindrical: MagLIF (ICF)

Metallic liner

Cylindrical: wire array Z pinch (non-ICF)

Roughly speaking, once wires have cooked away, the implosion begins
Backup: wire array ablation

Spherical: inertial confinement fusion (ICF) capsule

- DT gas
- DT fuel
- X-ray radiation (indirect drive)
- Laser (indirect drive)
- Ablator
- Implosion: $v \approx 3 \times 10^5$ m/s
- Stagnation: $R \approx 1$ mm
- Hot spot: $R \approx 40 \mu$m

Cylindrical: MagLIF (ICF)

- Metallic liner
- $B_z$, $B_\theta$, $j \times B$

Cylindrical: wire array Z pinch (non-ICF)

- Precursor plasma
- $B_\theta$
Backup: mass injection scheme

$R_0 = 1\, \text{cm}$

$L = 7\, \text{mm}$
Backup: Model ablation/implosion via mass injection

This idea has been used before:
Backup: mass injection parameters constrained by experiment

Each cell has mass $m$ and ablates according to

$$m = m_0 (I/I_0)^\alpha$$

determines when cell finishes ablating.

Determines when array starts to implode.

currently,$^*$

$\alpha = 1.4$,
determines distribution of prefill plasma.


mass is injected slowly ($v \sim 1e4$ m/s) at mass injection surface

$R_0 = 1$ cm

2502 ns

cell has cleared out

DENSITY

5.000e+00
3.750e+00
2.500e+00
1.250e+00
0.000e+00
Backup: mass injection parameters constrained by experiment

\[ \dot{m} = \dot{m}_0 \left( \frac{I}{I_0} \right)^\alpha (1 + \epsilon(z)) \]

spectral content determined by experimental histogram. Amplitude constrained by contrast ratio between streams.

\( R_0 = 1 \text{cm} \)

Cell has cleared out

\( \lambda = 0.51 \text{ mm} \)
\( \text{sd} (\lambda) = 0.096 \text{ mm} \)
Backup: 3D structure during implosion

C=3%, t=2518 ns
This is very different from a Bennett equilibrium.
t = -0.6 ns

3D simulation fluid flows

\[ t(ns) \]

0.00 0.05 0.10 0.15 0.20 0.25

\[ E_{\text{KE}} \ (\text{J}) \]

\[ K_{\text{E}} \ (\text{MJ}) \]

\[ I_{\text{MA}} / 100 \]

\[ I_{\text{E}} \ (\text{MJ}) \]

\[ p = 1 \times 10^{12} \ \text{J/m}^3 \]
Less evidence for vortices. This is sensible: we are sampling decreasing $p_{\text{ram}}$, which is insufficient to “turn the flow around”.
Back up: centrifugal force for $t > 0$

Now centrifugal force $\ll$ pressure gradient over much of the core.
Most of the core plasma is expanding outward, but notice the inflow of hot plasma into the core.
Effect of residual flows on energy transport

\[ \langle \rho \rangle (\text{kg/m}^3) \]

\[ \langle T_e \rangle (\text{K}) \]

-0.8 ns
\( t = -1.2 \text{ ns} \)
Effect of residual flows on energy transport

\[ \langle \rho \rangle (\text{kg/m}^3) \]

\[ \langle T_e \rangle (\text{K}) \]

-0.4 ns
-0.8 ns
t=-1.2 ns
On-axis $T_e$ is constant during compression

After $t=-1$ ns, $T_e(r=0)$ doesn’t increase--not good for HED/ICF.

Usually, plasma heats while compressing i.e. for adiabatic plasma

\[
p \sim \rho^\gamma
\]

\[
p = \frac{k_b}{m_i} \rho T \implies T \sim \rho^{\gamma-1}
\]

$\gamma \sim 1.3$ for W

\[
T \uparrow \text{as } \rho \uparrow
\]
On-axis $T_e$ is constant during compression

After $t=-1$ ns, $T_e(r=0)$ doesn’t increase--not good for HED/ICF.

Usually, plasma heats while compressing i.e. for adiabatic plasma

$$p \sim \rho^\gamma$$

$$p = \frac{k_b}{m_i} \rho T \implies T \sim \rho^{\gamma^{-1}} \quad \gamma \sim 1.3 \text{ for W}$$

$T \uparrow$ as $\rho \uparrow$

Of course, plasma is not adiabatic; thermal conduction could prevent $T$ from rising (recall radiation is off).

But a 1D estimate shows thermal conduction is too weak an effect.

Could convective flows explain this behavior?
Effect of convection on energy transport

During compression, we saw outflow of hot core plasma – cooling mechanism.

We apply the energy equation to a R=0.1 mm cylinder to study these convective effects.
Convective energy flow dominates during compression

Even during compression, convection of energy OUT is large. Combined with pdV cooling, it is responsible for keeping $T(r=0) \sim \text{const.}$

- Convection leads to enhanced thermal transport over thermal conduction
- However $\kappa$ in DT is larger: $\kappa \propto \frac{T^{5/2}}{Z \ln \Lambda}$
Backup: on-axis \( T_e \) is constant during expansion

After \( t=0.4 \) ns, \( T_e(r=0) \) doesn’t decrease.

Usually, plasma **cools** while expanding i.e. for adiabatic plasma

\[
T \sim \rho^{\gamma^{-1}} \quad \gamma \approx 1.3 \text{ for W}
\]

\[ \text{T} \downarrow \text{as } \rho \downarrow \]

Plasma is not adiabatic. Joule heating, thermal conduction, shock heating could all provide heating.

But again, estimates show these effects are **too weak** to counteract pdV cooling.

Could **convective** flows explain this behavior?
During compression, we saw outflow of hot core plasma – cooling mechanism. During expansion, we see inflow of hot material – heating mechanism.

We apply the energy equation to a R=0.1 mm cylinder to study these convective effects.
Backup: Convective energy flux dominates during expansion

- Even though plasma is expanding, inflow transports significant energy into the core. This is the reason $T_e(r=0)$ can stay constant during expansion.
- Convection leads to enhanced thermal transport over thermal conduction.

**Energy convection out (sink):**
$$-1.17 \times 10^{13} \text{ W}$$

**Energy convection in (source):**
$$7.7 \times 10^{12} \text{ W}$$

**pdV cooling (sink):**
$$-2.65 \times 10^{12} \text{ W}$$

**pdV heating (source):**
$$1.5 \times 10^{12} \text{ W}$$

**Shock heating (source):**
$$1.16 \times 10^{11} \text{ W}$$

**Joule heating (source):**
$$6.9 \times 10^{10} \text{ W}$$

**Thermal conduction (source):**
$$3 \times 10^{10} \text{ W}$$

$$\frac{\partial}{\partial t} \int_V \rho e dV = -\int_{v_r>0} \rho ev \cdot dS - \int_{v_r<0} \rho ev \cdot dS$$

$$-\int_{\nabla \cdot v>0} p \nabla \cdot v dV - \int_{\nabla \cdot v<0} p \nabla \cdot v dV$$

$$+ \int_V \tau : \nabla v dV + \int_V \frac{j^2}{\sigma} dV - \int_S q \cdot dS$$
Backup: determination of $R_s(t)$ from $\langle p(r) \rangle$
Backup: determination of $R_s(t)$ from $<p(r)>$

- $P_0=3e12$ J/m$^3$

- $R_s(t=0)$

- $t=0$ ns

- $P=3e12$ J/m$^3$
Backup: Justification for isothermal solution

$v(r)$ is linear. This allows particles to compress in unison, without shocks.

Also now finite $T$ is allowed. We consider the isothermal solution, motivated by the enhanced thermal transport seen in 3D simulation.
Stagnation is complete. All kinetic energy has converted to internal energy. The key parameter:

\[ \epsilon = \frac{\int \rho dV}{\int \frac{1}{2} \rho v^2 dV} \]

Initial thermal energy

Initial kinetic energy

\[ \frac{\rho_f}{\rho_i} = e^{1/\epsilon} \]
Backup: profile comparison between shockless solution and 3D simulation

3D profiles agree with theoretical fits for $r \leq 0.3$ mm
Backup: energy flow in a Z pinch