Beam Loss in Periodic Permanent Magnet Klystrons

M. Hess and C. Chen
MIT - Plasma Science and Fusion Center

Presented at
APS Snowmass
Snowmass, Colorado
July 1-21, 2001

*Research supported by AFOSR and DOE.
Outline of the Presentation

- Motivation
- Experimental Evidence of Beam Loss in PPM Klystrons
- Electrostatic Green’s Functions
- Previous Work at MIT on PPM Klystrons
- Bunched Pencil Beam Model
- Discovery of a Beam Confinement Criterion
- Comparison of Criterion to SLAC PPM Klystrons
- Conclusions
Motivation

- A major challenge in developing high intensity charged beam devices, such as high-power klystron tubes and high-intensity accelerators is how to eliminate the problem of beam loss.

- Possible mechanisms for beam loss include: beam halo formation, magnetic focusing errors, unstable beam-wall interactions, etc.

- In this talk, we will discuss bunched beam confinement and the beam-wall interaction as a possible mechanism for beam loss.
Experimental Evidence

- Pencil Beam Microwave Sources

- SLAC Periodic Permanent Magnet (PPM) Klystrons:
  - **50 MW-XL-PPM**: Measured 0.8% beam power loss (D. Sprehn, et al, Proc. LINAC98, p. 689, 1998)
  - **Klystrino**: Still under development (G. Scheitrum, 2000)
Advantages of Green’s Functions

- Green’s functions can provide a powerful technique for the modeling of high-power microwave sources and particle accelerators.
  - Both electrostatic and electromagnetic interactions with realistic boundary conditions can be modeled
  - Better resolution than PIC codes
  - Allows for analytical modeling of sources (e.g. klystrons)

- Using a Green’s function, we self-consistently model pencil thin bunched beams and annular bunched beams.
  - Find conditions for beam confinement
  - Find a mechanism for beam loss
Diagram of Pencil Beam Model

2-D Configuration

3-D Configuration

Electron Beam Halo Formation \[ \langle P_\theta \rangle = 0 \]
Summary of Beam Halo Simulation Results*

\[ \langle P_\theta \rangle = 0 \]

1.5% of Electrons in Halo

Beam Tunnel Radius

Equilibrium Beam Radius

halo radius

core radius

RF Output

Observation of Beam Loss in Klystron*

- **Basic Parameters**
  - **Beam Current**: 190 A, 190 A, 190 A
  - **Beam Voltage**: 464 kV, 464 kV, 464 kV
  - **Cathode Radius**: 2.86 cm, 2.86 cm, 2.86 cm
  - **Magnetic Field at Cathode**: N/A, 0.0 G, 6.9 G
  - **Beam Radius**: 2.38 mm, 2.05 mm, 2.38 mm
  - **Beam Tunnel Radius**: 4.7625 mm, 9.0 mm, 9.0 mm
  - **Total Tube Length**: 90 cm, 90 cm, 90 cm
  - **PPM Focusing Section Length**: 42 cm (~ 20 periods), 42 cm, 42 cm
  - **RMS Magnetic Field**: 1.95 kG, 1.95 kG, 1.95 kG

- **Experimental Testing Results**
  - 99.9% beam transmission without RF input;
  - 50 MW, 1.5 microsec at 55% efficiency with 0.8% beam power loss observed;
  - No beam loss predicted by particle-in-cell CONDOR simulations.

Green’s Function

\[ \nabla^2 G_{3D} = -\frac{4\pi}{r} \delta(r-r') \delta(\theta-\theta') \delta_L(z-z') \]

\[ \delta_L(z-z') = \sum_{n=-\infty}^{\infty} \delta(z-z'-nL) \]

\[ G_{3D}|_{r=a} = 0 \]

Definition

\[ \alpha = \frac{2\pi a}{L}, \quad \hat{r} = \frac{2\pi r}{L}, \quad \hat{r}' = \frac{2\pi r'}{L}, \quad \hat{z} = \frac{2\pi z}{L}, \quad \hat{z}' = \frac{2\pi z'}{L} \]

Solution

\[ G_{3D}(x; x') = \frac{2}{L} \sum_{n=-\infty}^{\infty} \sum_{l=-\infty}^{\infty} e^{ln(z-z')} e^{il(\theta-\theta')} \frac{1}{I_l(n\alpha)} \]

\[ \times \begin{cases} 
I_l(n\hat{r})[I_l(n\alpha)K_l(n\hat{r}) - I_l(n\hat{r}')K_l(n\alpha)] & , \quad (\hat{r} < \hat{r}') \\
I_l(n\hat{r}')[I_l(n\alpha)K_l(n\hat{r}) - I_l(n\hat{r})K_l(n\alpha)] & , \quad (\hat{r} > \hat{r}') 
\end{cases} \]
By Lorentz transforming to the beam rest frame, the problem becomes completely electrostatic.

Using the Green’s function to compute the induced surface charge on the wall, we calculate the electric field acting on each bunch due to the beam-wall interaction:

$$
\vec{E}^{\text{self}}(\hat{r}) = -\frac{4\pi N_b e}{\gamma^2_b L^2} \left[ \hat{r} \frac{\hat{r}^2 - \alpha^2}{\alpha^2 - \hat{r}^2} - 2 \sum_{n=1}^{\infty} K_0(n\alpha) I_0(n\hat{r}) \left( I'_0(n\hat{r}) - 2 \frac{\hat{r}^2 - \alpha^2}{\alpha^2 - \hat{r}^2} \right) - 4 \sum_{n=1}^{\infty} \sum_{l=1}^{\infty} K_l(n\alpha) I_l(n\hat{r}) I'_l(n\hat{r}) \right] \hat{e}_r
$$

2-D 3-D (Effect of Bunching)

$$
\alpha = \frac{2\pi a}{\gamma_b L}, \quad \hat{r} = \frac{2\pi r}{\gamma_b L}, \quad \gamma_b = \frac{1}{\sqrt{1 - \beta_b^2}}
$$
Beam-Wall Interaction (cont.)

- Transformation back to the laboratory frame yields:

\[
\vec{E}_{\text{lab}}^{\text{self}} = \gamma_b \vec{E}^{\text{self}} \\
\vec{B}_{\text{lab}}^{\text{self}} = -\gamma_b \vec{\beta}_b \times \vec{E}^{\text{self}}
\]

- These fields take into account the electromagnetic wakefields observed in the lab frame due to longitudinal motion of all the charges.
- The fields do not include effects from transverse motion.
The self-fields may be expressed in terms of potentials and incorporated into a single particle relativistic Hamiltonian:

\[
H = \sqrt{(c\vec{P} - QA_{lab}^{self} - QA_{ext})^2 + M^2 c^4 + Q\phi_{lab}^{self} - \frac{QE_{rf} \cos(kz - \omega t)}{k}}
\]

\[
\vec{E}_{lab}^{self} = -\vec{\nabla}\phi_{lab}^{self} \quad \vec{B}_{lab}^{self} = \vec{\nabla} \times \vec{A}_{lab}^{self}
\]

The rf traveling wave term is included to model fields in an rf accelerator.

For bunched beams in a klystron drift tube section, we may assume the rf term is zero.
Hamiltonian Analysis (cont.)

Generating Function:
\[ F(z, P_z', t) = (z - v_{ph} t) P_z' \]
\[ v_{ph} = \omega/k \]

Canonical Transformations:
\[ z' = z - v_{ph} t = \partial F/\partial P_z' \]
\[ P_z = P_z' = \partial F/\partial z \]
\[ H' = H + \partial F/\partial t \]

Decompose Energy:
\[ H' = H_{\parallel} + H_{\perp} \]
\[ H_{\parallel} >> H_{\perp} \]
\[ H_{\parallel} = \gamma_b M c^2 - v_{ph} P_z' - \frac{Q E}{k} \sin(k z') \]
\[ H_{\perp} = \frac{1}{2 \gamma_b M} \left[ P_r^2 + \left( \frac{P_\theta}{r} - \frac{Q}{c} A_{\theta}^{\text{ext}} \right)^2 \right] + \frac{Q \Phi^{\text{self}}}{\gamma_b^2} \]
Averaged Equations of Motion: Periodic $B$

- We assume that the transverse time scale is much slower than the longitudinal time scale through the PPM field

Using:

$$\langle H'_\perp \rangle = \frac{k_0}{2\pi} \int_0^{2\pi/k_0} dz H'_\perp, \quad A^\text{ext}_\theta = \frac{rB_0}{2} \cos[k_0 z]$$

$$\langle H'_\perp \rangle = \frac{1}{2\gamma_b M} \left( P_r^2 + \frac{P^2}{r^2} + \frac{Q^2 B^2_{\text{rms}} r^2}{4c^2} \right) + \frac{Q \phi^{\text{self}}}{\gamma_b^2}$$

**Radial Motion Equation:**

$$\frac{dr}{dt} = \frac{P_r}{\gamma_b m_e N_b}$$

**Radial Momentum Equation:**

$$\frac{dP_r}{dt} = \frac{1}{\gamma_b} \left[ \frac{P^2}{m_e N_b r^3} - \frac{e^2 N_b B^2_{\text{rms}}}{4m_e c^2} - \frac{N_b e}{\gamma_b} \frac{\partial \phi^{\text{self}}}{\partial r} \right]$$
Confined and Unconfined Electron Orbits

High Magnetic Field

Low Magnetic Field
Radial Phase Space Plots

“High Magnetic Field”

“Low Magnetic Field”

Unconfined Orbit

Confined Orbit
Confinement Criterion I \((P_\theta \neq 0)\)

Define:

\[
\omega_p^2 = \left( \frac{4\pi e^2}{m_e} \right) \left( \frac{N_b}{\gamma_b \pi a^2 L} \right), \quad \omega_c = \frac{eB_{\text{rms}}}{m_ec}
\]

\[
2\omega_p^2\omega_c^2
\]

Graph showing the relationship between \(2\omega_p^2/\omega_c^2\) and \(2P_\theta/m_e \omega_c a^2\) with different values of \(a/\gamma_b L\): infinity, 0.3, 0.15, 0.08.
The upper bound on the self field parameter when confined orbits exist occurs at $P_\theta = 0$. Analysis yields:

- **The 2-D case:**
  \[
  \frac{2\omega_p^2}{\omega_c^2} \leq 1 \quad \text{(Brillouin Density Limit)}
  \]

- **The 3-D case:**
  \[
  \frac{2\omega_p^2}{\omega_c^2} \leq \frac{1}{1 + \sum_{n=1}^{\infty} \frac{n\alpha}{I_0(n\alpha)I_1(n\alpha)}}
  \]
The critical electron density for a bunched beam is considerably lower than the usual Brillouin density.
Periodic Permanent Magnet (PPM) Focusing Klystron Amplifier*

*from SLAC Web Site.
## Table of Parameters for SLAC PPM Klystrons

<table>
<thead>
<tr>
<th>PARAMETER</th>
<th>50 MW-XL</th>
<th>75 MW-XP</th>
<th>KLYSTRINO</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f$ (GHz)</td>
<td>11.4</td>
<td>11.4</td>
<td>95</td>
</tr>
<tr>
<td>$I_b$ (A)</td>
<td>190</td>
<td>257</td>
<td>2.4</td>
</tr>
<tr>
<td>$\gamma_b$</td>
<td>1.83</td>
<td>1.96</td>
<td>1.22</td>
</tr>
<tr>
<td>$B_{rms}$ (T)</td>
<td>0.20</td>
<td>0.16</td>
<td>0.29</td>
</tr>
<tr>
<td>$a$ (cm)</td>
<td>0.48</td>
<td>0.54</td>
<td>0.04</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>0.75</td>
<td>0.77</td>
<td>1.15</td>
</tr>
<tr>
<td>$\frac{8c^2 I_b}{\omega_c^2 a^2 I_{A,exp}}$</td>
<td>0.19</td>
<td>0.28</td>
<td>0.35</td>
</tr>
<tr>
<td>$\frac{8c^2 I_b}{\omega_c^2 a^2 I_{A,cr}}$</td>
<td>0.238</td>
<td>0.244</td>
<td>0.366</td>
</tr>
</tbody>
</table>

\[
L = \frac{\beta_b c}{f}
\]

\[
\alpha = \frac{2\pi a}{\gamma_b L} = \frac{2\pi af}{c\sqrt{\gamma_b^2 - 1}}
\]

\[
I_b = N_b e f
\]

\[
2\omega_p^2 = \frac{8c^2 I_b}{\omega_c^2 a^2 I_A}
\]

\[
I_A = \frac{\gamma_b \beta_b m_e c^3}{e} \approx 17kA\gamma_b \beta_b
\]
Comparison of Theory to Experiment

\[ \alpha = \frac{2\pi a}{\gamma b L} \]

- a. 50 MW-XL-PPM
- b. 75 MW-XP
- c. Klystrino
Magnetic Field Limit for Strong Bunching

In the limit of strong bunching:

$$\alpha = \frac{2\pi af}{c\gamma_b \beta_b} \leq 3.0$$

$$\frac{2\omega_p^2}{\omega_c^2} \leq \frac{1}{1 + \sum_{n=1}^{\infty} \frac{n\alpha}{I_0(n\alpha)I_1(n\alpha)}} \approx \frac{\alpha}{\pi}$$

which implies

$$B_{rms} \,(kG) > 142 \left( \frac{Q}{1 \, nC} \right)^{1/4} \left( \frac{a}{1 \, cm} \right)^{-3/2}$$
Conclusions

- Modeled bunched pencil beam with a Green’s function technique

- Established confinement criterion on the space-charge limit for both types of bunched beams

- Space-charge limit for pencil beams agrees well with SLAC PPM klystron parameter, 75 MW-XP may observe greater beam loss since it is operating above limit.

- Possible solutions to beam loss in 75 MW-XP: open beam pipe radius, increase magnetic field (hybrid focusing system)
Models for Klystron Interactions

- Equilibrium beam transport is treated at a kinetic level.
  - Electrostatic description;
  - Determination of detailed electron distribution in the absence of rf fields.

- The axial current oscillations on the electron beam and rf power transfer are treated at a fluid level.
  - Electromagnetic description;
  - Estimation of deviations from the beam equilibrium in the presence of the rf fields.

- The transverse dynamics is treated at a kinetic level, using a Green’s function based, two-dimensional self-consistent model.
  - Electrostatic description;
  - Determination of detailed electron distribution in the presence of the rf fields.