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# Experimental validation of similarity in high-temperature plasmas

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**Abstract**
The scaling of energy transport with dimensionless parameters has been measured in high-temperature plasmas with the goals of guiding theory and predicting energy confinement in future fusion devices. Validation of this approach requires demonstration of similarity in plasmas with identical dimensionless parameters but very different physical parameters. Within measurement uncertainties, the heat diffusivities and global energy confinement exhibit similarity in high-confinement regimes on the DIII-D and JET tokamaks and in low-confinement regimes on the DIII-D and Alcator C-Mod tokamaks.

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Dimensionless parameter scaling has been a standard tool in both theoretical and experimental physics for many years [1]. The fundamental concept is that the behaviour of classes of physical systems can be determined from the scaling of the phenomenon of interest with the dimensionless parameters which appear in the governing equations. This is especially fruitful for complex systems where the governing equations cannot be directly solved. A familiar example is the behaviour of fluid systems where a single dimensionless parameter, the Reynolds number, characterizes the behaviour of flows.

One of the aims of this work is to understand the scaling of energy transport with device size in high-temperature magnetized plasmas. Early theoretical work applying scaling with dimensionless parameters [2, 3] focused on the scaling implications of various simplified descriptions of plasma behaviour as a means of validating these models experimentally. More recently, it was noticed that the present-day high-temperature plasmas differ from plasmas expected in a deuterium–tritium fusion power plant in only a single dimensionless parameter—the gyroradius normalized to a linear device dimension ($\rho_*$) [4]. It was suggested that carefully crafted experiments varying $\rho_*$ on a single device could provide the necessary information to determine the minimum size of a fusion power plant despite the fact that the size of the plasmas in the experiment do not change! Experiments to measure the scaling of energy confinement with $\rho_*$ have been reported on several fusion devices [5–8]. However, the necessary validation of the applicability of dimensional analysis to plasma energy transport has not been performed. An essential element of the validation is to demonstrate that plasmas with widely different physical parameters (including size), but identical dimensionless parameters, have similar normalized energy transport, i.e., they exhibit similarity. This report details the first experimental demonstration of similarity in local energy transport for the purpose of this validation.

The fundamental assumption is that the plasmas studied are governed by the Vlasov–Maxwell system of equations including the effects of collisions [2, 3]. It is further assumed that the plasma can be described by the lowest three velocity space moments of the equations, which is equivalent to assuming local thermodynamic equilibrium on the timescale of energy transport. On this timescale, only gradients perpendicular to the magnetic field lines need to be considered, because the plasma equilibrates along field lines much more rapidly. The plasma geometry is constructed to be identical except for scale and to consist of nested toroidal surfaces of constant magnetic flux. Finally, quasi-neutrality is assumed to hold for processes which cause energy transport. This assumption means the displacement current is neglected in the Ampére law, or equivalently, perturbations on the Debye length scale are ignored [3]. The above assumptions result in evolution equations for the density $n$, temperature $T$, and angular rotation $\vec{\omega}$ of the plasma, and the two components of the magnetic field $\vec{B}$ in the flux surface as the description of the plasma. The plasma moves freely in the toroidal direction due to axisymmetry, so only the toroidal angular rotation $\omega_T$ need be considered. This leaves five local variables to combine into dimensionless parameters ($n$, $T$, $\omega_T$, $B_T$, $B_p$).

There is no unique choice of dimensionless parameters; however, it is advantageous to choose combinations which commonly arise in theoretical models or which have known physical limits [2]. In this paper, the following combinations will be used: the normalized gyroradius $\rho_*(\propto \sqrt{T}/B_T a)$, the ratio of the kinetic to magnetic pressure $\beta(\propto nT/B_T^2)$, the normalized collision frequency $\nu^\dagger(\propto na/T^2)$, the Mach number $M(\propto \omega_T a/\sqrt{T})$, and the safety factor $q(\propto B_T/B_p)$. The heat diffusivity $\chi$ is the quantity for which the size scaling is desired. The heat diffusivity can be written as

$$\chi = \chi_B F\{\rho_*, \beta, \nu^\dagger, M, q, g\}, \qquad (1)$$

where $\chi_B \equiv T/eB_T$ and is known as the Bohm diffusion coefficient. Here, $\chi_B$ serves as a dimensionally correct reference diffusivity; it could be replaced by any quantity with the appropriate dimensions. The quantity $g$ symbolizes the requirement that the geometry of the plasmas must also be matched. Only local quantities have been discussed so far, but it is likewise possible to consider the global energy confinement:

$$\omega_c \tau_{th} \propto B\tau_{th} \propto G\left\{\langle\rho_*\rangle, \langle\beta\rangle, \langle\nu^\dagger\rangle, \langle M\rangle, \frac{aB}{I}, g\right\}, \qquad (2)$$

where $I$ is the plasma current. The normalization of the thermal confinement time $\tau_{th}$ to the cyclotron frequency $\omega_c$ is again conventional.

The principle of similarity states that two plasmas which have identical dimensionless parameters must have the same normalized transport $\chi/\chi_B$ or normalized confinement $B\tau_{th}$ despite having different physical parameters. In order to match the local dimensionless parameters $\rho_*$, $\beta$, $\nu^\dagger$, $M$, and $q$ simultaneously in both plasmas, the following quantities must be matched: $na^2$, $Ta^{1/2}$, $\omega_T a^{5/4}$, $B_T a^{5/4}$, and $B_p a^{5/4}$. If these are matched, then $\chi/\chi_B \propto \chi a^{-3/4}$. For matching global parameters, the formal volume average of these parameters is not calculated. Instead, quantities which are simpler to evaluate are used: $\bar{n}a^2$, $\beta_{th}$, $Ba^{5/4}$, and $q_{95}$. Here $\bar{n}$ is the line-averaged density, $\beta_{th}$ is the ratio of the thermal pressure to the magnetic pressure, $B$ is the vacuum toroidal field evaluated at the geometric centre $R$ of the outermost flux surface, and $q_{95}$ is the safety factor evaluated at the flux surface with 95% of the total poloidal flux. Parameters related to the plasma geometry such as aspect ratio $R/a$ and the vertical elongation $\kappa$ are used to check the match of the plasma shape.

Starting first with a global parameter comparison, the similarity conditions can be matched to better than 4% in both the high confinement (H)-mode and the standard confinement (L)-mode regimes typical of auxiliary heated tokamak plasmas. Cross-section views of the three tokamaks exployed in the similarity test are shown in figure 1 to scale. For the H-mode comparison, plasmas in the JET and DIII-D tokamaks have virtually exact matches for dimensionless matching criteria described above for very different engineering parameters (table 1). While the actual confinement times differ by more

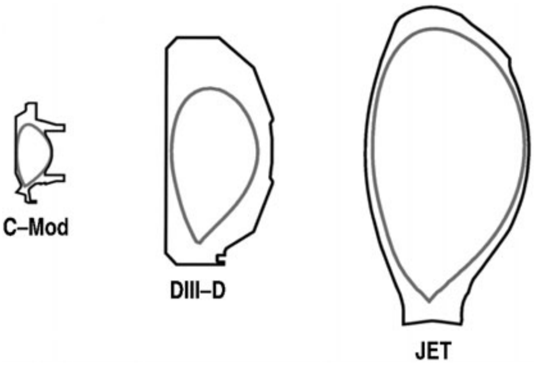


**Figure 1.** Cross-section elevations of the Alcator C-Mod, DIII-D, and JET tokamaks (from left to right, respectively). The physical size of the plasmas used in these similarity studies vary by a factor of 4.4.

**Table 1.** Comparison of the global engineering and dimensionless matching criteria for the H-mode and the L-mode similarity tests.

| | H-mode | | L-mode | |
|---|---|---|---|---|
| | JET | DIII-D | DIII-D | C-Mod |
| Engineering | | | | |
| $B$ (T) | 1.05 | 2.1 | 1.61 | 5.22 |
| $a$ (m) | 0.97 | 0.56 | 0.57 | 0.22 |
| $\bar{n}$ ($10^{19}$ m$^{-3}$) | 2.1 | 6.4 | 2.6 | 18.0 |
| $P_{abs}$ (MW) | 3.4 | 5.6 | 1.9 | 3.2 |
| $\tau_{th}$ (s) | 0.26 | 0.122 | 0.079 | 0.027 |
| Dimensionless | | | | |
| $R/a$ | 3.0 | 3.0 | 3.06 | 3.07 |
| $Ba^{5/4}$ | 1.01 | 1.01 | 0.79 | 0.79 |
| $\bar{n}a^2$ | 2.0 | 1.99 | 0.82 | 0.85 |
| $\beta_{th}$ (%) | 1.7 | 1.7 | 0.52 | 0.51 |
| $q_{95}$ | 3.6 | 3.6 | 3.7 | 3.6 |
| $B\tau_{th}$ | 0.27 | 0.26 | 0.127 | 0.141 |

than a factor of 2, the normalized confinement times match to better than 4%. In the L-mode comparison, again the global criteria for plasmas in the DIII-D and Alcator C-Mod tokamaks are well matched. The normalized confinement time matches to about 10%, while the actual confinement time varies by almost a factor of 3. The larger mismatch in the normalized confinement could be due to a systematic difference in the method used to determine the input power in the two tokamaks. The potential systematic error is in the direction to improve the agreement. Global analysis of a larger set of DIII-D/JET similarity pairs under different operating conditions is shown in figure 2 along with the two comparisons given in table 1. Good agreement in normalized confinement over a range of ~2.5 indicates that the similarity principle at the level of global confinement is valid over a considerable range in operational space.

The good agreement of the global analysis motivates the more stringent test of comparison of the local heat diffusivities. The experimental value of the diffusivity is obtained by solving the energy continuity equation, which can be written schematically as

$$\frac{3}{2}n\frac{\partial T}{\partial t} + \nabla \cdot Q = S, \qquad (3)$$

where $Q$ is the heat flux and $S$ represents the net value of various sources and sinks of energy. The heat diffusivity

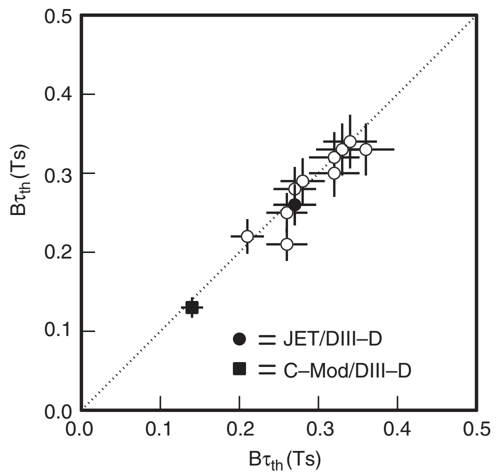


**Figure 2.** Comparison of normalized global thermal energy confinement time $B\tau_{\mathrm{th}}$ of JET (●) or Alcator C-Mod (■) discharges on the horizontal axis with $B\tau_{\mathrm{th}}$ for matched similar DIII-D discharges (vertical axis). The filled symbols are the two pairs of discharges compared in table 1. The dotted line shows the expected locus for pairs with perfect similarity. Fixed uncertainties of 10% are shown.

$\chi$ is defined by the relation $Q \equiv -n\chi\nabla T$. The density and temperature profiles are measured, and a combination of model calculations and measurements are employed to evaluate $S$. For the evaluation of $\chi$ for all three tokamaks, the TRANSP code [9] is used to avoid systematic differences in the implementation of the solution of equation (3). The densities are sufficiently high in the Alcator C-Mod and DIII-D discharges discussed here that separation of the plasma into distinct electron and ion fluids with individual diffusivities is not possible. The change in the magnitude of the collisional energy exchange between electrons and ions as the measurements are varied within their uncertainties would substantially alter the inferred diffusivities. Treating the plasma as a single fluid removes this source of uncertainty. When temperature is discussed, the one-fluid temperature $T \equiv (p_{\mathrm{e}} + p_{\mathrm{i}})/(n_{\mathrm{e}} + n_{\mathrm{i}})$ is used, where $p$ is the pressure and the subscripts e and i refer to the separate electron and ion quantities, respectively. The ions are treated as a single species with an effective charge state $Z_{\mathrm{eff}} \equiv (\Sigma_j n_j Z_j)/n_{\mathrm{e}}$ where the sum is over all charge states of all ions present.

For the DIII-D/JET comparison of H-mode discharges, well-matched normalized density and temperature profiles were obtained as shown in figure 3. Both plasmas were heated by means of injection of neutral particle beams. It was assumed that the toroidal and poloidal fields were matched as well as possible by matching the global parameters. The match was evaluated over the range of normalized minor radius $\rho = 0.30$–$0.75$. At smaller and larger radii, the profiles are strongly affected by magnetohydrodynamic instabilities (sawteeth and ELMs, respectively) which are not properly resolved with the diagnostic time resolution available for this analysis. In principle, the similarity concept should hold in these regions also. The agreement of normalized density and temperature profiles is quite remarkable. The profiles of rotation and $Z_{\mathrm{eff}}$ are less well matched.

The normalized diffusivities for the two plasmas at the time of matched density and temperature agree within the

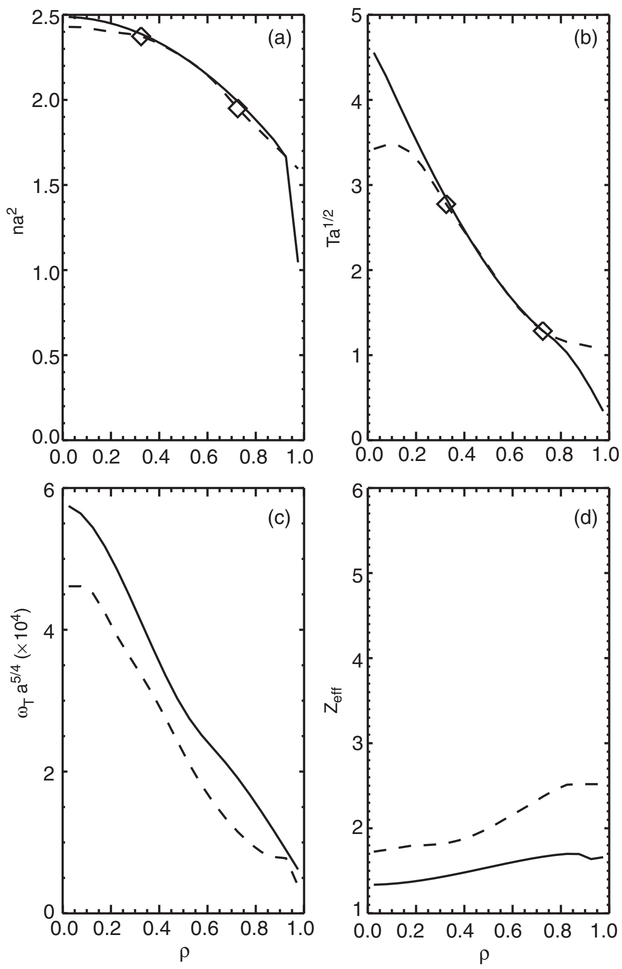


**Figure 3.** Comparison of the radial profiles of the normalized density, temperature, rotation, and effective charge profiles for DIII-D (——) and JET (- - - -). The normalizations are defined in the text. The profiles were selected on the basis of best matching the normalized density and temperature for the radial range between the open diamonds. The radial coordinate $\rho$ is the square root of the toroidal flux normalized to the boundary value.

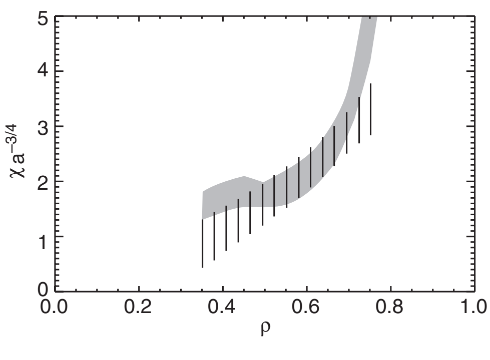


**Figure 4.** Radial profiles of the normalized one-fluid diffusivities for DIII-D (dashed region) and JET (shaded region). The bands indicate the estimated uncertainty due to random error as explained in the text.

displayed uncertainty (figure 4). The uncertainty shown is evaluated by taking a population standard deviation of the diffusivities for the time history. This is likely a slight overestimate of the random uncertainty since any slow trend

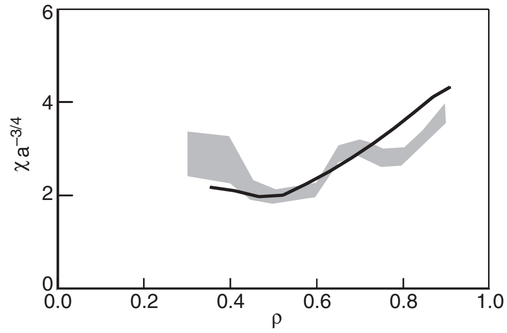


**Figure 5.** Radial profiles of the normalized one-fluid diffusivities for DIII-D (——) and Alcator C-Mod (shaded region).

will be included in the standard deviation. Note that the diffusivities match closely in the region where the normalized densities are well matched (figure 3(*a*)). This is a general trend in the dataset—at other times, the normalized densities are less accurately matched and the diffusivities are also systematically different. The uncertainties in the diffusivities due to systematic error are difficult to quantify. The largest potential source of systematic error in both machines is the heating deposition profile. There are no direct measurements of the deposition, and any deviation from the model directly affects the evaluation of the diffusivity. Indirect evidence in the DIII-D case indicates that the error can be no larger than a 30% reduction. This is of the same magnitude as the random uncertainty. Therefore, the data support a local validation of similarity in energy confinement, but with larger uncertainty than the global comparisons.

A preliminary result indicating the validity of dimensionless parmeter scaling using DIII-D and JET H-mode data has been previously reported [10]. This result was not considered by the authors to be an appropriate validation due to a lack of ion temperature, rotation, and dilution data from JET. This lack of data was a critical point since the assumption of equal electron and ion temperatures might not be valid in the low-density regime in which JET must be run to match the dimensionless parameters of DIII-D. This work has the appropriate data from JET and an analysis of the uncertainties due to random errors in the input data.

Local analysis of the DIII-D and Alcator C-Mod discharges also indicates similarity in energy confinement under L-mode conditions. The normalized diffusivities are shown in figure 5 for both tokamaks. The DIII-D plasma was heated by injection of neutral particle beams, while the Alcator C-Mod plasma was heated by resonant radiofrequency waves. The uncertainties in the Alcator C-Mod diffusivity are estimated in the same way described above. Only a single time in the DIII-D discharge was analysed, so no uncertainty estimate is shown. It is expected to be of the same magnitude as that shown in figure 4.

In conclusion, the principle of similarity has been successfully demonstrated for tokamak energy transport in both well-known confinement regimes (L-mode and H-mode). This indicates that physical phenomena at the scale of the Debye length and other effects not considered, such as atomic physics, do not play a dominant role in the transport of energy across the magnetic field in these plasmas. Validation of similarity is a key benchmark for the application of $\rho_*$ scaling to estimating confinement in next-step devices. The device size in this demonstration spans from the centrepoint (DIII-D) downward a factor of 2.6 (Alcator C-Mod) and upward a factor of 1.8 (JET). However, the appropriate physics measure of size is $1/\rho_*$, the number of gyroradii spanning the minor radius. In concert, DIII-D and JET can scan $\rho_*$ by a factor of 3 in H-mode [11], while JET lies within a factor of 4 of these future devices [11–13]. Therefore, $\rho_*$ scaling between these two devices can make a significant contribution to predictions of confinement performance. Planned upgrades of the Alcator C-Mod auxiliary heating system would allow $\rho_*$ scaling experiments in H-mode over a range similar to the extrapolation to future experiments. The validity of similarity here establishes an experimental basis for the use of dimensionless parameter scaling for energy confinement projections.

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