Full-wave Electromagnetic Field Simulations of Lower Hybrid Waves in Tokamaks

John C. Wright
M. Brambilla and P. Bonoli

In collaboration with the Wave – Particle Interaction SciDAC
Wave-Particle SciDAC group

D. B. Batchelor – ORNL, Principal Investigator
L. A. Berry, M. D. Carter, E. F. Jaeger – ORNL Fusion Energy
E. D’Azevedo – ORNL Computer Science and Mathematics (OASCR – SSAP)
C. K. Phillips, H. Okuda – PPPL
P. T. Bonoli, John Wright – MIT
D. N. Smithe – Mission Research Corp.
R. W. Harvey – CompX
D. A. D’Ippolito, J. R. Myra – Lodestar Research Corporation
M. Brambilla – IPP Garching
Geometric optics describes some LH phenomena, but not others

- **Caustic formation**, $k_r=0$
  [Pereverzev NF 1992, Barbato 1983]

- **Spectral gap**, not all effects captured by ray tracing
  - Scattering off density fluctuations, requires wave-kinetic eqn.
    [Bonoli PF 1981]
  - Parametric decay, non-linear mechanisms
    [Porkolab 1978, Cesario PRL 2004]
  - **Diffraction**, needs fullwave or higher order WKB
    [Wright IAEA 2004, Pereverzev NF 1992]
  - **Toroidal upshift**, insufficient in some devices
    [Bonoli PF 1987, Baranov 1980]
Fullwave LH model

- Wave Equation
  \[ \nabla \times \nabla \times E = \frac{\omega^2}{c^2} \left\{ E + \frac{4\pi i}{\omega} (J^P + J^A) \right\} \]

- Dielectric Tensor for LH freq.
  \[ \Omega_{ci}^2 \ll \omega^2 \ll \Omega_{ce}^2 \]
  - Strongly magnetized electrons
  - Unmagnetized ions
  Maxwellian Dist used.

- 3 modes for bicubic
  - Fast and slow waves
  - Ion plasma wave

Maxwellian Dist used.
Associated Disp Rel:

\[ P_6 n_\perp^6 + P_4 n_\perp^4 + P_2 n_\perp^2 + P_0 = 0 \]

\[ P_0 = \epsilon_\parallel \left[ (n_\parallel^2 - \epsilon_\perp)^2 - \epsilon_{xy}^2 \right] \]
\[ P_2 = (\epsilon_\perp + \epsilon_\parallel)(n_\parallel^2 - \epsilon_\perp) + \epsilon_{xy}^2 \]
\[ P_4 = \epsilon_\perp \]
\[ P_6 = - \left( \frac{3}{8} \beta_e \frac{\omega^2}{\Omega_{ce}^2} + \frac{3}{2} \beta_i \frac{\Omega_{ci}^2}{\omega^2} \right) \]
LHRF Physics Regime—cold ions and magnetized electrons

- Frequency range is intermediate of ion and electron cyclotron frequencies:

\[ \Omega_{ci} \ll \omega \ll \Omega_{ce} \]

\[ k_{\perp} \approx -\frac{\varepsilon_{\parallel}}{\varepsilon_{\perp}} k_{\parallel} \]

\[ \varepsilon_{\perp} = 1 + \frac{\omega_{pe}^2}{\Omega_{ce}^2} - \frac{\omega_{pi}^2}{\omega^2} \approx 1 \]

\[ \varepsilon_{\parallel} = 1 - \frac{\omega_{pe}^2}{\omega^2} - \frac{\omega_{pi}^2}{\omega^2} \approx -\frac{\omega_{pe}^2}{\omega^2} \]

\[ \rightarrow k_{\perp} \approx \frac{\omega_{pe}}{\omega} k_{\parallel} \]

- Fourth order DR (with fast and slow LH waves) predicts an accessibility criteria

\[ n_{\parallel} > n_a \equiv \frac{\omega_{pe0}}{\Omega_{ce0}} + \epsilon_{\perp 0}^{1/2} \]

- \( \omega / \omega_{\text{lh}} > 2 \), no ion-plasma wave

Typical Parameters:

- \( B_0 = 4.5 \, T, \, D^+ \)
- \( f_0 = 4.6 \, \text{GHz} \)
- \( n_{\parallel} = 2.5, \, n_a = 2 \)
- \( n_{e0} = 2 \times 10^{20} \, m^{-3} \)
- \( \omega / \Omega_{CD} \approx 125 \)
- \( k_{\perp} \approx 66 \, \text{cm}^{-1} \)
The TORIC decomposition results in a block tridiagonal stiffness matrix, where each block is \((6*\text{Nm})^2\), with \(3*\text{Nr}\) blocks.

Each large block is distributed across the processors and its inverse written to disk using an out-of-core technique, greatly extending the size of problem that can be done.

Lower hybrid runs can take as many as 5000 cpu-hours. In cases with caustics where \(k_r\) vanishes:

\[
\frac{m}{r} \approx k_\perp \approx \frac{\omega_p}{\omega} k_\parallel \to m \approx 1000
\]

All runs are done on the 50 processor MIT-PSFC theory cluster, MARSHALL (see Sherwood Mon 5:30 special cluster session).
Create an edge absorption case with a caustic

- Wave is accessible in a narrow region.
- Fast LH waves are launched and mode convert to slow LH waves.
- Waves are trapped between inner caustic and outer cutoff.

Plasma Parameters:
- D(5% H), n_||=1.5, f=4.6 GHz
- B_0 = 5.3 T, T_e=3.5 keV, T_i=2.0 keV
- n_e(0)=1.5x10^{20} m^{-3}, I=1 MA
RayTracing Results

- ACCOME, initial ray
- Ray penetrates after reflections.
- Damps on hotter core electrons after many reflections.
- We see a different picture with a full wave calculation
There is a suggestion of resonance cones in the full wave field patterns. Loci of LH reflections can be seen in the full wave field patterns, similar to raytracing field structure.
Power Depositions radially different

- ACCOME has damping penetrating to the center and starting inside the caustic radius.

- TORIC shows power deposition is confined to a narrow region of $r/a = [0.65, 0.83]$.

The difference may be related to the downshifting of $v_{\text{phase}}$ by diffraction, which is high at the caustic - evidence for full wave enhancement of filling of the spectral gap.
Full-Wave diffraction broadens spectrum rapidly

- For raytracing, the local $n_\parallel$ evolves to 2.5 on the high field side from 1.5 at the antenna due primarily to geometric effects of major radius position.
- The distribution of $n_\parallel$ on flux surfaces shows a significant upshift from an averaged launched $n_\parallel$ of 2 to >4 in the middle of the annulus at r/a=0.75.
Fullwave LH model

- Wave Equation
  \[ \nabla \times \nabla \times E = \frac{\omega^2}{c^2} \left\{ E + \frac{4\pi i}{\omega} (J^P + J^A) \right\} \]

- Dielectric Tensor for LH freq.
  \[ \Omega^2_{ci} \ll \omega^2 \ll \Omega^2_{ce} \]
  - Strongly magnetized electrons
  - Unmagnetized ions

- 3 modes for bicubic
  - Fast and slow waves
  - Ion plasma wave

Maxwellian Dist used.
Associated Disp Rel:

\[ P_0 n_0^0 + P_4 n_\perp^4 + P_2 n_\perp^2 + P_0 = 0 \]

\[
\begin{align*}
P_0 &= \varepsilon_\parallel \left[ (n_\parallel^2 - \varepsilon_\perp)^2 - \varepsilon_{xy}^2 \right] \\
P_2 &= (\varepsilon_\perp + \varepsilon_\parallel)(n_\parallel^2 - \varepsilon_\perp) + \varepsilon_{xy}^2 \\
P_4 &= \varepsilon \left( \frac{3}{8} \beta_e \frac{\omega^2}{\Omega^2_{ce}} + \frac{3}{2} \beta_i \frac{\omega^2}{\Omega^2_{ci}} \right) \\
P_6 &= - \left( \frac{1}{8} \beta_e \frac{\omega^2}{\Omega^2_{ce}} + \frac{3}{2} \beta_i \frac{\omega^2}{\Omega^2_{ci}} \right)
\end{align*}
\]
TORIC has been optimized for FWLH calculations

- Need to eliminate ion plasma wave
  - Algebraically eliminate $E_\psi$ in code and drop $dE_\psi/d\psi$ from eqn for $E_\psi$
  - This decouples $E_\psi$, leaving only two modes, back substitute to get all three components.

- **TORIC solver modified for LHRF:**
  - Unmagnetized ions, strongly magnetized electrons
  - 4th order system, two waves – fast and slow LH
  - New boundary conditions at plasma wall to impress $E_{||}$
  - LH BCs use waveguide model:

- This version is being parallelized.
  The following cases less severe resolution requirements and use the serial version for cases with:

\[
\begin{align*}
\frac{c}{\omega} \left[ (\nabla \times E)_{[\eta, \zeta]} \right]_{\psi} &= \frac{4\pi i}{c} J^S \left[ -\sin \Theta_w, \cos \Theta_w \right] \\
\left[ E^{(m)}_{[\eta, \zeta]} \right]_{\psi} &= -e^{-im\theta_0} e^{\frac{-im\Delta_\theta + 1}{4m^2 \Delta_g^2 - \pi^2}} \left[ \sin \Theta_w, \cos \Theta_w \right]
\end{align*}
\]
Alcator C study – wave propagation

- Alcator-C experiment had high field (10T) and lower hybrid CD (1.5 MW).
- Waves are accessible to the center for this case.
- Rays form caustics near center.
- Note similarity in field structures.

Plasma Parameters:
Resolution (240Nr x 127 Nm)
H plasma, n∥=1.5, f=4.6 GHz
B_0 = 10 T, T_e=1.8 keV, T_i=1.0 keV
n_e (0)=0.5×10^{20} m^{-3}, I=170 kA
Spectral evolution and power

- Power locations comparable
- Larger upshift seen in raytracing
  - Need more resolution for fullwave?
Alcator C-Mod

- New C-Mod LH system (Parker Mon A-07)
- High $n_{\|}$ leads strong damping.

No energy inside $r/a<0.3$.
May be resolved with serial code.

Parameters: Same as first case, with $n_{\|}=4$, $f=4.6$ GHz
Resolution (240 Nr x 127 Nm)
Conclusions

- Use of parallel full wave algorithm provides a new tool for investigating lower hybrid wave physics.
- Full-wave studies are now elucidating important physics effects (focusing and diffraction) not easily included in geometrical optics treatments.
  - Diffraction is especially important if strong caustics are formed by the lower hybrid waves.
  - May provide ubiquitous mechanism for bridging the spectral gap.
- This work has used Maxwellian distributions for ions and electrons. Next steps include coupling the CQL3D electron Fokker-Planck code to TORIC.