Challenges in self-consistent full wave simulations of lower hybrid waves

John C. Wright

P. T. Bonoli, J-P. Lee - MIT
E. J. Valeo, C. K. Phillips - Princeton
R. W. Harvey - Comp-X

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Participants in the RF-SciDAC


P.T. Bonoli, J.C. Wright, H. Kohno, A.S. Richardson, J-P. Lee

R.W. Harvey, A.P. Smirnov, N.M. Ershov

M. Choi, D. D’Ippolito, J. Myra

D. Smithe, T. Austin

C.K. Phillips, E. Valeo, N. Gorelenkov, H. Qin

M. Brambilla, R. Bilato

R. Maggiora, D. Milanesio

M. Choi, D. D’Ippolito, J. Myra

Lodestar
Motivation

- Lower hybrid generates efficient current, and is the only option in outer plasma.
- Assess full wave effects – the computational resources needed to do this now exist.
- Electric field is needed for
  - Direct evaluation of the quasilinear diffusion accounting for phase interference,
  - rf-sheath interactions at the wall,
  - coupling to Newtonian and Monte-Carlo calculations of plasma response (diffusion, distribution fn evolution).
- Implementation of boundary conditions is well defined.
Conductivity Relation - LHRF

LHCD Regime: $\Omega_{ci}^2 << \omega^2 << \Omega_{ce}^2$ and $\omega \geq 2\omega_{LH} ; \omega_{LH} \sim (\Omega_{ci} \Omega_{ce})^{1/2}$

- Unmagnetized ions
- Strongly magnetized electrons $[(k\perp \rho_e)^2 << 1]$

Wave equation is sixth order with two propagating modes, one damped:

- Mode converted ion plasma wave is not propagative, so drop sixth order term
- Electrostatic LH “slow wave” branch
- Electromagnetic LH “fast wave” branch

$$P_4 n_\perp^4 + P_2 n_\perp^2 + P_0 = 0$$

Wave lengths are very short:

$$\lambda_\perp \approx (\omega/\omega_{pe})\lambda_\parallel \approx 1\text{mm}$$

Predicts an accessibility criterion:

$$n_\parallel > n_a \equiv \frac{\omega_{pe}}{\Omega_{ce}} + S^{1/2}$$

Bonoli PoF 1982
LH Absorption physics

- Parallel refractive indexes are geometrically up-shifted as waves propagate to smaller major radius. Poloidal asymmetries can cause spread in m spectrum.

\[ n_\parallel = \frac{c}{\omega} \left( \frac{m}{q} + n \right) / R \]

- Quasilinear damping occurs at \( \omega/(k_\parallel v_{te}) \approx 3 \) =>

\[ n_\parallel \approx \frac{5.7}{\sqrt{T_e[k eV]}} \]

This also sets poloidal resolution.

so lower temperatures require higher \( n_\parallel \) for damping.

- Higher parallel refractive indexes are more accessible to the interior of the plasma but also damp at lower temperatures=larger minor radii.

- Current drive scales as \( 1/n_{e0} n_\parallel^2 \) and \( n_{\text{acc}}[n_{e0},B] \) sets minimum \( n_\parallel \)

=> operation in weak damping regime for \( T_{e0} < 16 \text{keV} \)
Approaches to solution

- WKB expansions: ray tracing (GENRAY, ACCOME) and beam tracing (LHBEAM) are asymptotic approximations to the wave equation, $k \sim \nabla$ required. => can be problems with boundaries

- Full Wave (TORLH): solves Maxwell's equations directly, yields the electric field.

\[ \nabla \times \nabla \times E = \frac{\omega^2}{c^2} \left\{ E + \frac{4\pi i}{\omega} (J^P + J^A) \right\} \]
TORIC Full Wave Code

- TORIC [Brambilla PPCF 1999] was developed with an FLR model for the plasma current response, $J^p$, for ion cyclotron waves and recently extended with an asymptotic form for lower hybrid waves.

- The antenna is modeled as a current sheet, $J^A$ for ICRF and as the mouth of a wave guide with imposed $E_\parallel$ for LH.

- It solves Maxwell's equations for a fixed frequency (Helmholtz problem) assuming toroidal symmetry in a mixed spectral-finite element basis. This is the physical optics solution.

$$\nabla \times \nabla \times E = \frac{\omega^2}{c^2} \left\{ E + \frac{4\pi i}{\omega} (J^P + J^A) \right\} \quad \leftrightarrow \quad E(x) = \sum_m E_m(\psi) \exp(i m \theta + i n \phi)$$
The pressure driven term $\sigma$ is responsible for the ion plasma LH branch. In regimes of experimental interest it is nearly vanishing.

- We drop this term and solve only for the fast and slow LH waves.
- The dielectric assumes zero FLR electrons and unmagnetized ions (non-Maxwellian effects in Im $P$ retained).
- Residue gives Im $P$, principle value gives Re $P \sim Z_0$.

\[
\nabla \times \nabla \times E = \mathcal{S}E_\perp + iD(b \times E_\perp) + PE_\parallel b + \nabla_\perp(\sigma \nabla_\perp \cdot E)
\]
Scaling/Convergence study

- Progression from 511->1023->2047
- Power deposition broadens at each step
- 18hrs on 2048 cpus on Franklin for $N_m = 2047$
Scaling/Convergence study

- Fully converged on Maxwellian distribution at 2047 poloidal modes.
Single pass is well converged

- Single pass damping on a Maxwellian plasma is converged
- Little interference evident

Power is very localized in single pass.

<- Spectrum is converged at 1023 modes

$n_\parallel = -2.55$, $n_{e0}=7e19$ m$^{-3}$, $T_{e0} = 10$ keV, $B_0=4$T
Cross-code validation

Alcator C.
≈10^6 mesh pts in each simulation. \( n_\parallel = -2.5, \ B_0 = 8 \text{T}, \ n_e = 5 \times 10^{19} \text{ m}^{-3}, T_e = 5 \text{keV}. \)

Each approach as advantages and difficulties:

- CPU-hrs: COMSOL-LH 13, TORICLH 80, AORSA 32k (ray tracing ~ minutes) but wall clock times for all three are comparable given number of processors used.
- COMSOL: 2D elements can model wall and separatrix region; sparse matrix scales well [PoP Sep 2009]
- The plasma dielectric:
  - COMSOL-LH: requires real space dielectric formulation, doesn't have algebraic \( k_\parallel \)
  - TORIC-LH: FLR truncation efficient for LH
  - AORSA: All Orders treatment most general, can handle fast ion interactions
First ever 3D LH full wave simulation

- Sum 100 nphi:

\[
E_z(r, \theta, \phi) = \sum_{n_{\text{tor}}} \frac{P_{\text{LH}}(n_{\text{tor}}) \times P_{\text{abs}}(n_{\text{tor}})}{P_{\text{abs}}(n_{\text{tor}}) \sum_{n_{\text{tor}}}(P_{\text{ant}}(n_{\text{tor}}) \times P_{\text{abs}}(n_{\text{tor}}))} e^{i(n_{\text{tor}} + \phi_{\text{ant}}(n_{\text{tor}}))} E_{z,n_{\text{tor}}}(r, \theta)
\]

Alcator C
f=4.6 GHz, \( n_{||0} = 2.1 \)
H plasma, 8T, 600kA, 5x10^{13} cm^{-3}, 3.8 keV
(240,127,100)x6=10^{7} \text{DoF}
Model plateau [2.5,8]\( v_{th} \)
J-P. Lee with VisIt
Movie of 3D LH full wave

- Animation of poloidal slices of $\text{Re}(E_\psi)$ at advancing values of phi.
- Observe the motion of the resonance cone.
- The wave fields are fully damped as the cone reaches the hotter plasma center.
**CPU requirements are substantial**

- Typical resolutions of 1000x1023.
- The stiffness matrix, $A$ is block tridiagonal with the blocks being $(6 \times N_m)^2$.
- Current approach distributes 3 blocks over all processors and uses the serial Thomas algorithm.
- Existing parallel tridiagonal solvers do not distribute the blocks.
New solver has 3D decomposition

- **Current Solver**
  \[
  \begin{align*}
  L_i \cdot \vec{x}_{i-1} + D_i \cdot \vec{x}_i + R_i \cdot \vec{x}_{i+1} &= \vec{y}_i \\
  D_{i+1} &= D_{i+1} - L_{i+1} \times D_i^{-1} \times R_i
  \end{align*}
  \]
  \((6N_m) \times (6N_m)\)

- **New Solver**
  \[
  Ax = \begin{bmatrix}
  a_1 & b_1 \\
  c_2 & a_2 & b_2 \\
  & c_3 & a_3 & b_3 \\
  & & \ddots & \ddots & \ddots \\
  & & & c_n & a_n & b_{n-1} \\
  \end{bmatrix}
  \begin{bmatrix}
  x_1 \\
  x_2 \\
  x_3 \\
  \vdots \\
  x_n \\
  \end{bmatrix} = \begin{bmatrix}
  r_1 \\
  r_2 \\
  r_3 \\
  \vdots \\
  r_n \\
  \end{bmatrix} 
  \equiv r.
  \]

- **Serial** (Radial direction \([i=1..N_\psi]\): Thomas Algorithm)
- **2-D Parallel** (Poloidal \(m\) modes: Scalapack matrix calculation \((6N_m) \times (6N_m)\))

- **1-D Parallel** (Radial direction: combination of Divide-and-Conquer and Odd-even cyclic reduction Algorithms) \(\Rightarrow \# \text{P1 groups}
  

  +

  2-D Parallel (Poloidal \(m\) modes: Scalapack matrix calculation )
  \(\Rightarrow \#P2*P3\) processors
  \(= 3-D\) grid \((P_{\text{tot}}=P1*P2*P3)\)
Strong scaling of old and new solvers to 16k processors

- Time shown only for solver step. Fill in step has slope $\sim -1$. 2-4 x faster than Thomas solver.
- Old solver (in red) saturated because of complete domain decomposition, not communication.
Weak damping requires self-consistent $f(v)$ for damping

- The poloidal power spectrum shows lack of convergence at outer flux surfaces,
- and magnification of unit amplitude applied field.
Simulation model is a coupled full wave and Fokker-Planck system

CQL3D (Harvey 1992 IAEA)

TORIC-LH (Wright 2004 CPC, 2009 PoP)

Dielectric for LH uses zero FLR electrons and unmagnetized ions. Calculates the rf fields and the bounce avg. quasilinear flux.

- Simulations use EFIT reconstructed magnetic equilibria.
- Electron distribution functions from iteration with a Fokker-Planck code are used for dielectric – nonlinear.
Hard X-ray gives indirect measure of LH CD

- HXR camera measures bremsstrahlung emissions from electrons accelerated by LHCD.
- Better for simulation comparison than current profile – harder to measure, longer time response.
Non-relativistic iteration is not sufficient

- Broad plateau formation observed.
- Pitch angle scattering creates high energy tails $\sim 500$ keV.
- Synthetic HXR from CQL3D/TORICLH is narrower and weaker than experimental measurement. Total current is 300 kA vs 700 kA in experiment. What effects are missing?

TORIC results $\times 4$

Compared to experiment
Relativistic effects in dielectric

- Relativistic consistency:
  - Dql from TORIC is fully relativistic,
  - CQL3D evolves the relativistic distribution function,
  - Formulation of parallel dielectric response for general relativistic non-Maxwellian retains the principle value in the form of the Maxwellian Z-fn, Im part has resonance along a hyperbolic line in u-space.

\[
Im \chi_{zz} = \frac{\omega_{pe}^2}{\omega^2} n_\parallel \int 2\pi u_\perp du_\perp u_\parallel J_0^2 \left( k_\perp u_\perp / \Omega_e \right) \frac{\partial f}{\partial u_\parallel}
\]

\[
u_\perp^2 = (n_\parallel^2 - 1)u_\parallel^2 - 1
\]
Relativistic effects on Dql

- Inclusion of relativistic effects introduces the hyperbolic resonance condition:

\[ \omega = k_{||} v_{||} \Rightarrow \omega = k_{||} u_{||}/\gamma \Rightarrow (n_{||}^2 - 1)u_{||}/c^2 - u_{\perp}^2/c^2 = 1 \]

\[ B = \sum_{m,m'} \frac{u_{||}^4}{\gamma^2 v_{\phi}^{(m)} v_{\phi}^{(m')}} J_0^2 \left( k_{\perp} u_{\perp} / \Omega_{e0} \right) E_{||}^{(m)} E_{||}^{*(m')} e^{i\theta(m-m')} \delta(\omega - k_{||} v_{||}) \]
Using the Alcator C-Mod shot #1060728011.1100. We calculate and compare the HXR spectrum with a synthetic diagnostic.

Using the fields generated from a Maxwellian dielectric we get agreement in strength but not detailed shape profile. Code now matches experimental current magnitude.
Summary

- We have developed a tool that for the first time can produce self-consistent simulations of lower hybrid current drive in toroidal geometry using the full wave approach and 3D Fokker-Planck.
- Spectral broadening effects due to diffraction and poloidal coupling are included in the model.
- The full wave calculation yields the electric fields directly.
- Wave fronts are calculated properly near caustics and cutoffs. This is may be important in the multipass damping regime.
- Improvements in algorithms and computation platforms have both been important in making 3D full wave LH possible.
- Cross code and experimental validation have been essential in development.
Backup slides
The launched waves will not damp until the phase velocity slows down to about $3v_{te}$.

Full wave solutions include diffraction effects for this upshift of $n_{||}$. 
A brief history of TORIC

- **FISIC (100x63)** – Brambilla NF 1986, F77, introduced basic algorithm, reduced order for $E_{||}$
- **TORIC (240x127)** – Brambilla NF 1996, F77+F90, completely restructured, full solution
- **TORIC out-of-core (480x255)** – extends problem size
- **TORIC parallel (980x1023)** – F90+MPI+ScaLaPACK, extend problem size, reduces run time
- **TORIC-LH** – Wright CiCP 2004
- + **Fokker-Planck** – Wright PoP 2009
- + **3D processor mesh (2000x2047)**
Comparison with ray tracing, weak damping

- Maxwellian damping in both cases.
- Good agreement in power deposition locations.
- In full wave, we see the field amplitudes are magnified by “cavity effect” [Q~200] from unit field at the guide.

Alcator C-Mod
\( n_e = -1.55, B_0 = 5.3T, \)
\( \n_e = 7 \times 10^{19} \text{ m}^{-3}, T_e = 2.3\text{keV}. \)
HXR Camera

Hard X-ray camera designed and built to image fast electron bremsstrahlung emission produced by LHCD.

HXR Camera Features:
- Views poloidal cross section at the mid-plane from B-port
- 5x5x2 mm CZT detectors to measure photons from 20-250 keV
- 32 channels giving spatial resolution of ~1.5 cm
- Compact size
- Integrated and modular electronics
- High signal count rate
- Fast digitization and software processing techniques
- ~3 cm Pb gamma shielding
- Adjustable mount fixed to igloo
Comparison of $f_e(v)$

Contour plot of distribution function

Ray tracing

Relativistic distribution functions.
Summary, cont.

- Improvements in algorithms and computation platforms have both been important in making 3D full wave LH possible.
- Cross code and experimental validation have been essential in development.
- Integration with Fokker-Planck and using experimental inputs is manual and error prone – a consistent database driven method is needed to improve this.
Ray Tracing Models for Wave Propagation

- Toroidal ray tracing code GENRAY:

- Integrates the ray equations of geometrical optics.

\[
\begin{align*}
\frac{dx}{dt} &= -\frac{\partial D/\partial k}{\partial D/\partial \omega} \\
\frac{dk}{dt} &= +\frac{\partial D/\partial x}{\partial D/\partial \omega} \\
\frac{dP}{dt} &= -2\gamma P
\end{align*}
\]

Ray tracing shows evidence of focusing at caustic surfaces that can be treated using full-wave or advanced ray tracing methods.

Reflections are sensitive to edge gradients.
Simulation model is a coupled full wave and Fokker-Planck system

CQL3D (Harvey 1992IAEA)

\[ \frac{\partial}{\partial t} (\lambda f_0) = \nabla u_0 \cdot \Gamma u_0 \]

Bounce averaged Fokker-Planck code that solves for new distribution function from RF quasilinear flux, provides synthetic HXR diagnostic

TORIC-LH (Wright 2004CPC,2009PoP)

\[ \nabla \times \nabla \times E = \frac{\omega^2}{c^2} \left\{ \frac{4\pi i}{\omega} \left( J^P + J^A \right) \right\} \]

\[ E(x) = \sum_m E_m(r) \exp(imp + i2\phi) \]

\[ k_\parallel = (mB \cdot \nabla \theta + n_\phi B \cdot \nabla \phi) / B \]

\[ J^P(r) = \sum_m \sigma_c \left( k_\parallel^m, r \right) \cdot E_m(r) \]

Dielectric for LH uses zero FLR electrons and unmagnetized ions.

- Simulations use EFIT reconstructed magnetic equilibria.
- Electron distribution functions from iteration with a Fokker-Planck code are used for dielectric.
Outline

- Introduction to Lower Hybrid waves
- Modeling LH waves
  - Ray tracing
  - Full Wave
  - Comparisons between approaches
- Numerics of Full Wave simulations
- Full Wave effects and Non-thermal electrons
- Summary and Conclusions
Calculated following full wave treatment in Wright NF1997.

Bounce averaged, Larmor cutoff $[ J_0(k_{\perp \rho}) ]$ at high perpendicular energies. Non-relativistic.

Note the symmetry in the trapped region.

$$D_{ql} \sim \sum u_{||}^4 E_{||}^{(m1)} E_{||}^{(m2)} e^{\delta(\omega - k_{||} v_{||})}$$

$$\gamma^2 v_{\phi}^{(m1)} v_{\phi}^{(m2)}$$

$$u_{||} = \gamma v_{||}, \gamma = 1$$
Effect of non Maxwellian damping

- Convergence achieved in two iterations.
- Note the reduction in peak amplitude.
- Fields are less space filling now.
Effect of non Maxwellian damping

- Spectrum is converged.
- Power is a bit broader.

Energy in poloidal modes plotted for selected flux surfaces.

Power density and Poynting flux. Power is normalized to the total power.