Self-consistent full wave simulations of lower hybrid waves

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2
Outline

• Introduction to Lower Hybrid waves
• Modeling LH waves
  – Ray tracing
  – Full Wave
• Comparisons between approaches
• Full wave effects, non-thermal electrons, relativistic effects
• Summary and Conclusions
Motivation

- Predicted HXR Spectra Agree Well with Measured Spectra – Except Narrower [Bonoli PoP2008]

- Assess full wave effects – the computational resources needed to do this now exist.

- Electric field is needed for
  - Direct evaluation of the Kennel-Engelmann quasilinear diffusion accounting for phase interference,
  - sheath interactions,
  - coupling to Newtonian and Monte-Carlo calculations of plasma response (diffusion, distribution fn evolution).

- Implementation of boundary conditions is well defined.
LHCD Regime: $\Omega^2_{ci} \ll \omega^2 \ll \Omega^2_{ce}$ and $\omega \geq 2\omega_{LH}$

- Unmagnetized ions
- Strongly magnetized electrons $[(k_{\perp} \rho_e)^2 \ll 1]$

Wave equation is sixth order with two propagating modes, one damped:
- Mode converted ion plasma wave is not propagative, so drop sixth order term
- Electrostatic LH “slow wave” branch
- Electromagnetic LH “fast wave” branch

$$P_4 n_{\perp}^4 + P_2 n_{\perp}^2 + P_0 = 0$$

Wave lengths are very short:

$$\lambda_{\perp} \approx (\omega/\omega_{pe}) \lambda_{||} \approx 1\text{mm}$$

Predicts an accessibility criterion:

$$n_{||} > n_\alpha \equiv \frac{\omega_{pe}}{\Omega_{ce}} + S^{1/2}$$

Bonoli PoF 1982
LH Absorption physics=>weak damping characterizes present tokomaks

- Parallel refractive indexes are geometrically up-shifted as waves propagate to smaller major radius. Poloidal asymmetries can cause spread in m spectrum.
  \[ n_\parallel = \frac{c}{\omega} \left( \frac{m}{q} + n \right) / R \]

- Quasilinear damping occurs at \( \omega/k_\parallel v_{te} \sim 3 \Rightarrow \)
  \[ n_\parallel \approx \frac{5.7}{\sqrt{T_e}[keV]} \]
  so lower temperatures require higher \( n_\parallel \) for damping.

- Higher parallel refractive indexes are more accessible to the interior of the plasma but also damp at lower temperatures=larger minor radii.

- Current drive scales as \( 1/n_e n_\parallel^2 \) and \( n_{acc}[n_e,B] \) sets minimum \( n_\parallel \)

  \[ \Rightarrow \text{operation in weak damping regime for } T_{e0} < 16\text{keV} \]
Approaches to solution

- **WKB expansions**: ray tracing (GENRAY, ACCOME) and beam tracing (LHBEAM) are asymptotic approximations to the wave equation, $k \gg$ required. $\Rightarrow$ can be problems with boundaries.

- **Full Wave (TORIC-LH)**: solves Maxwell's equations directly, yields the electric field.

\[ \nabla \times \nabla \times \mathbf{E} = \frac{\omega^2}{c^2} \left\{ \mathbf{E} + \frac{4\pi i}{\omega} (\mathbf{J}^P + \mathbf{J}^A) \right\} \]

\[
\frac{dx}{dt} = -\frac{\partial D}{\partial k} \frac{\partial D}{\partial \omega} \\
\frac{dk}{dt} = +\frac{\partial D}{\partial x} \frac{\partial D}{\partial \omega} \\
\frac{dP}{dt} = -2\gamma P
\]
TORIC Full Wave Code

• **TORIC** [Brambilla 1999] was developed with an FLR model for the plasma current response, \( J^P \), for ion cyclotron waves and recently extended with an asymptotic form for lower hybrid waves.

• The antenna is modeled as a current sheet, \( J^A \) for ICRF and as the mouth of a wave guide with imposed \( E_\parallel \) for LH.

• It solves Maxwell's equations for a fixed frequency (Helmholtz problem) assuming toroidal symmetry in a mixed spectral-finite element basis. This is the physical optics solution.

\[
\nabla \times \nabla \times \mathbf{E} = \frac{\omega^2}{c^2} \left\{ \mathbf{E} + \frac{4\pi i}{\omega} \left( J^P + J^A \right) \right\} \quad \leftrightarrow \quad \mathbf{E}(x) = \sum_m \mathbf{E}_m(\psi) \exp(ism + in\phi)
\]
Simulation model is a coupled full wave and Fokker-Planck system

CQL3D (Harvey 1992IAEA)

Bounce averaged Fokker-Planck code that solves for new distribution function from RF quasilinear flux, provides $f(v)$

TORIC-LH (Wright 2004CPC, 2009PoP)

Dielectric for LH uses zero FLR electrons and unmagnetized ions. Calculates the RF fields and the quasilinear flux.

$\Rightarrow f_e(v)$ and $E(x)$ are self-consistent

Simulations use EFIT reconstructed magnetic equilibria.

Electron distribution functions from iteration with a Fokker-Planck code are used for dielectric.
The pressure driven term $\sigma$ is responsible for the ion plasma LH branch. In regimes of experimental interest it is nearly vanishing.

We drop this term and solve only for the fast and slow LH waves.

The dielectric assumes zero FLR electrons and unmagnetized ions (thermal damping in $P$ retained).

This reduced system is much less stiff and faster to solve.

Electron FLR terms discovered to cause SCK like numerical instability [Swanson PF 1981], because $k_\perp$ in $S,D$, and $\sigma$ must be consistent. Can drop because of negligible size.

$$\nabla \times \nabla \times \mathbf{E} = S \mathbf{E}_\perp + iD(\mathbf{b} \times \mathbf{E}_\perp) + PE_\parallel \mathbf{b} + \nabla_\perp (\sigma \nabla_\perp \cdot \mathbf{E})$$
Removal of small FLR e⁻ terms yields stable solution

Strong absorption $n_\parallel = 3.6$

Weak absorption $n_\parallel = 1.55$

- After zeroing out FLR electron terms fields from the antenna into the plasma are apparent.
- Interference is significant in low $n_\parallel$ case.
Full wave code approaches

- **COMSOL-LH** with 2D FE
- **TORIC-LH** with 1D FE/1D Spectral
- **AORSA** with 2D spectral

Each approach has advantages and difficulties:

- **CPU-hrs**: COMSOL-LH 13, TORIC-LH 80, AORSA 32k (ray tracing ~ minutes) but wall clock times for all three are comparable given number of processors used. (2hrs)
- **COMSOL**: 2D elements can model wall and separatrix region; sparse matrix scales well
- **The plasma dielectric**:
  - COMSOL-LH requires real space dielectric formulation, doesn't have algebraic $k_{||}$
  - TORIC-LH needs care in treating FLR truncation
  - AORSA – All Orders treatment most general, can handle fast ion interactions

**Example Cases**

- **Alcator C**.
  - 10^6 mesh pts. $n_{\perp} = -2.5$, $B_0 = 8$T, $n_{e0} = 5 \times 10^{19}$ m^{-3}, $T_{e0} = 5$keV.

- **Alcator C-Mod.**
  - 512x512, $n_{\perp} = -4.0$, $B_0 = 2$T, $n_{e0} = 6 \times 10^{19}$ m^{-3}, $T_{e0} = 4$keV.
Single pass is well converged with linear damping (Maxwellian)

- Single pass damping on a Maxwellian plasma is converged.
- Little interference is evident.

Power is very localized in single pass.

Poloidal spectrum on labeled flux surfaces

<- Spectrum is converged at 1023 modes

\[ n_{\parallel} = -2.55, \quad n_{e0} = 7 \times 10^{19} \text{ m}^{-3} \]
\[ T_{e0} = 10 \text{ keV}, \quad B_0 = 4 \text{T} \]
Comparison with ray tracing in weak damping limit

- Maxwellian damping in both cases.
- Good agreement in power deposition locations.
- But, field amplitudes are magnified by “cavity effect” \([Q \sim \omega/\gamma \sim 200]\)

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**Alcator C-Mod**

- \(n_\parallel = -1.55\), \(B_0 = 5.3\) T,
- \(n_{e0} = 7 \times 10^{19} \text{ m}^{-3}\), \(T_{e0} = 2.3\) keV.
Weak damping requires quasilinear $f_e(v)$ for self-consistent treatment.

* The poloidal power spectrum shows lack of convergence at outer flux surfaces,
* Also magnification of unit amplitude applied field.
$\langle D_{ql} \rangle_{\text{bounce}}$ for FP coupling

- Calculated following full wave treatment in Wright NF1997.
- Bounce averaged, Larmor cutoff $[J_0(k_\perp \rho_i)]$ at high perpendicular energies. Non-relativistic.
- Note the symmetry in the trapped region.

$$D_{ql} \sim \sum u_{\parallel}^4 E_{\parallel}^{(m1)} E_{\parallel}^{*(m2)} e^{i\theta(m1-m2)} \delta(\omega-k_{\parallel}^{(m1)} v_{\parallel})$$

$$\gamma^2 v_\phi^{(m1)} v_\phi^{(m2)}$$

$$u_{\parallel} = \gamma v_{\parallel} , \gamma = 1$$
Effect of non Maxwellian damping

- Convergence achieved in two iterations.
- Note the reduction in peak amplitude.
- Fields are less space filling now.
Effect of non Maxwellian damping

- Spectrum is converged.
- Power is a bit broader.
Example of QL tail formation, HXR spectrum from TORICLH/CQL3D

- Broad plateau formation observed.
- Pitch angle scattering creates high energy tails $\sim 500$ keV
- Synthetic HXR from CQL3D/TORICLH is narrower and weaker than experimental measurement
- $\Rightarrow$ What effects are missing?
Future Research

- More physics in calculation of $D_{qI}$
  - Inclusion of 3-D non-linear effects through multiple toroidal modes of antenna.
  - Relativistic effects.
  - Non-linear broadband effect in $D_{qI} \Rightarrow$ study with Newtonian $D_{qI}$ evaluation, e.g. Harvey this meeting with ions.

- Study the relative role of poloidal mode coupling (non-constancy of $m$) versus diffraction in observed spectral broadening.

- Application: synthetic PCI for LH on Alcator C-Mod (requires electric fields):
Relativistic effects on Dql

- Allowing for relativistic effects, $\gamma > 1$, $u \neq v$.

$D_{ql} \sim \sum u_\parallel^4 E_\parallel^{(m1)} E_\parallel^{*(m2)} e^{i\theta(m1-m2)} \delta(\omega-k_\parallel^{(m1)} v_\parallel)$

$= \gamma^2 V_\Phi^{(m1)} V_\Phi^{(m2)}$

- Main effect is transformation of the resonance condition, which depends only on electron velocity, not momentum. Becomes hyperbolic.

$\omega = k_\parallel v_\parallel \Rightarrow \omega = k_\parallel u_\parallel / \gamma \Rightarrow (n_\parallel^2 - 1) u_\parallel / c^2 - u_\perp^2 / c^2 = 1$
Summary

- We have developed a tool that for the first time can produce self-consistent simulations of lower hybrid current drive in toroidal geometry using the full wave approach and 3D Fokker-Planck.
- Spectral broadening effects due to diffraction and poloidal mode coupling are included in the model.
- The full wave calculation yields the electric fields directly.
- Wave fronts are calculated properly near caustics and cutoffs. This is may be important in the multipass damping regime.
References

Comparison of $fe(v)$

Relativistic distribution functions.