Fullwave coupling to a 3D antenna code using Green’s function formulation of wave-particle response

Three dimensional impedance calculations

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The impedance ($Z$) is the ratio between the voltage ($V$) and the current ($I$) or between the electric ($E$) and magnetic ($B$) field.

$$V = ZI \text{ or } \mathbf{E}_t = \mathbf{Z} \cdot \mathbf{B}_t$$

- It combines the dissipative and reactive properties of a system. $Z = R - iX$
- The admittance $Y$ is its inverse. $\leftrightarrow \leftrightarrow^{-1}$
  $$Y = \mathbf{Z}$$
- In two or three dimensions, we speak of the surface admittance matrix.

- In a discrete system, $S$, the admittance is related to the discrete Green’s function, $G_{m,m'} : \rightarrow SG_{m,m'} = \delta_{m,m'}$
  $$G_{m,m'} = S^{-1} \delta_{m,m'}$$

- Note that $S^{-1}$ is only a function of the system and not the boundary conditions. Total work is less than $4 \times$ a single solution with full boundary conditions. (Much less for small problems)

- After building up $G$, we can construct the impedance and the weighted reconstructed solution at only the cost of some matrix-vector products.
Adaptations of Topica and Toric for Coupling

- Set metal (E-field) boundary conditions at last flux surface in Toric.
- Transformation of Topica basis functions into the Toric space (POLITO).
- Making Toric export Z matrix calculated at LFS in file.
- Calculate reaction integrals of the aperture weight functions.
- Topica solution using impedance from Toric defines loading for each $E_{m,n}$. This is combined to get total loading and fields consistent with that loading in postprocessing step.

CAD rendering of C-Mod antenna used in Topica next to Toric full wave simulation.
Using **TORIC** and **TOPICA** codes [ **TOPICA** (TORino Polytechnic Ion Cyclotron Antenna) code is a numerical suite aimed at the performance prediction and analysis of plasma-facing antennas. It is capable of handling real-life 3D antenna geometries. We can find the impedance weighted wave solution as well as the surface fields on the antenna.

**TORIC** is run with a single \((m, n)\) excitation of a component of \(E_\eta, E_\xi\) at the plasma surface and the reactive magnetic field components are measured.

The admittance, \(\vec{Y}\), is defined as \(\vec{B} = \vec{Y}\vec{E}\) for the surface components of \(\vec{B}\) and \(\vec{E}\) where,

\[
\vec{Y} = \begin{pmatrix}
\vec{Y} \eta \eta & \vec{Y} \eta \zeta \\
\vec{Y} \zeta \eta & \vec{Y} \zeta \zeta 
\end{pmatrix}
\]
**The TORIC full wave model**

- **TORIC** solves with a discretization of electric field is Fourier in flux surface and Hermite finite elements in radial dimension

\[ E(x) = \sum_{m} E_m(r) \exp(i m \theta + i n \phi) \]

- in the Maxwell’s frequency domain equations for plasma dispersion model for \( \vec{\sigma} \)

\[ \nabla \times \nabla \times E = \frac{\omega^2}{c^2} \left\{ E + \frac{4\pi i}{\omega} \left( J^P + J^A \right) \right\} \]

\[ J^P = \vec{\sigma} [f_0(x, v_\perp, v_\parallel)] \cdot E \]

This results in a block tridiagonal stiffness matrix, where each block is \((6N_m)^2\), with \(3N_r\) blocks.

- Simulations can be run on one or several processors. For 3D calculations with modest poloidal mesh requirements, most efficient is to have each \(n_\phi\) mode on a separate processor, as there is no coupling along that dimension.
For unitary drive, the admittance is simply the value of the magnetic field responses. Peak at edges of spectrum indicates more resolution is needed for this case - response peaks near drive term at $m = 5$. 
TOPICA proposes to study a realistic antenna with a high level of geometric details (solid, rounded edges, etc.) including an accurate and realistic plasma description into the model. It uses the method of moments to formulate the matrix system.

\[
\begin{pmatrix}
-G_{11} & G_{12} - T_B \\
-G_{21} & G_{22} - G_P
\end{pmatrix}
\begin{pmatrix}
\sqrt{Z_0}I \\
-[K]/\sqrt{Z_0}
\end{pmatrix}
= \begin{pmatrix}
[E]/\sqrt{Z_0} \\
\sqrt{Z_0}[H]
\end{pmatrix}
\]

where $G_{nn}$ are the vacuum responses and $G_P$ is the plasma response related to the admittance matrix calculated by TORIC. Coupling model has the advantage that a given vacuum or plasma response, once calculated can be used with another to recalculate the loop currents and the plasma fields.
The coupling model assumes a coherent plasma antenna surface for this initial phase.

This collaboration will permit realistic antenna models for ICRF and LH full-wave studies.

With the addition of a triangular mesh for the vacuum region instead of the extended flux surface geometry now used, **TORIC** could model non-conforming antenna placement.
**Next Steps**

- **TOPICA** would have to be interfaced with a new set of basis functions compatible with the triangular basis.

- A new version of **TOPICA**, **TOPLA**, is being developed to model waveguide structures (the present version models current straps), that will permit the modelling of lower hybrid antennas.

- **TORIC** has recently been extending to model lower hybrid wave as well, so this would enable 3D modelling of the coupled lower hybrid antenna.
REFERENCES AND FURTHER READING

Numerical simulation of ion cyclotron waves in tokamak plasmas.

Nonthermal Particle and Full-Wave Diffraction Effects on Heating and Current Drive in the ICRF and LHRF Regimes.

Full-wave Electromagnetic Field Simulations of Lower Hybrid Waves in Tokamaks.
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