Analysis of Lower Hybrid Current Drive Scenario on Alcator C-Mod with a Full Wave Code

John C. Wright\textsuperscript{1} P. T. Bonoli\textsuperscript{1} M. Brambilla\textsuperscript{2} R. W. Harvey\textsuperscript{3} C. K. Phillips\textsuperscript{4} H. Okuda\textsuperscript{4}

\textsuperscript{1}\text{Plasma Science and Fusion Center, MIT}
\textsuperscript{2}\text{IPP-Garching}
\textsuperscript{3}\text{CompX}
\textsuperscript{4}\text{PPPL}

Oct 2005 APS-DPP at Denver

\textbullet Work supported under the RF-Scidac grant.
ABSTRACT

The newly installed lower hybrid system on Alcator C-Mod are to be used to drive current and shape the current profile in the plasma. The deposition location is sensitive to the launched parallel refractive index, \( n_\parallel \). The experiment can vary \( n_\parallel \) between 2 and 3. We use the full wave lower hybrid code, TORLH (TORIC code modified for LH simulations), with non-Maxwellian electrons and the Ehst-Karney parameterized adjoint model for current drive, to predict the variation with \( n_\parallel \) of the current drive deposition location in the plasma for a reference target plasma. We will discuss the importance of including full-wave effects such as focusing and diffraction in the calculation of the LH power deposition. In addition, we will discuss the role of 2D velocity space effects in the nonthermal electron distribution function on the LH power deposition.
LHRF waves have simple dispersion relation

For LHRF Physics Regime we use cold (unmagnetized) ions and magnetized electrons.

- No FLR ion effects in unmagnetized limit:
  \[(k_{\perp}\rho_i)^2 \rightarrow \infty\] (See Phillips, QP1.00037 this session)

- No electron FLR effects because of strong magnetization: \[(k_{\perp}\rho_e)^2 \ll 1\]

Frequency range is intermediate of ion and electron cyclotron frequencies:
\[
\Omega_{ci} \ll \omega \sim \omega_{LH} \ll \Omega_{ce}
\]
where
\[
\omega_{LH} = \frac{\omega_{pi}}{\sqrt{1 + (\omega_{pe}/\Omega_{ce})^2}},
\]

The conductivity in the lower hybrid limit is given by:

\[
\vec{\sigma} \cdot \vec{E} =
\]

\[
S \vec{E}_{\perp} + D (\vec{b} \times \vec{E}_{\perp}) + P E_{\zeta} \vec{b}
\]

\[
S \approx 1 + \frac{\omega_{pe}^2}{\Omega_{ce}^2} - \frac{\omega_{pi}^2}{\omega^2} \approx 1
\]

\[
D \approx -\frac{\omega_{pe}^2}{\omega^2} \frac{\omega}{\Omega_{ce}} + \frac{\omega_{pi}^2}{\omega^2} \frac{\Omega_{ci}}{\omega} \approx -\frac{\omega_{pe}^2}{\omega_{ce}}
\]

\[
P = 1 - \frac{\omega_{pe}^2}{\omega^2} - \frac{\omega_{pi}^2}{\omega^2} \approx \frac{\omega_{pe}^2}{\omega^2}
\]
For wavelength estimation, look at slab dispersion.

Consider the lower hybrid dispersion relation corresponding to the conductivity:

\[ P_4 n_\perp^4 + P_2 n_\perp^2 + P_0 = 0 \]
\[ P_4 = S \]
\[ P_2 = (S + P)(n_\parallel^2 - S) + D^2 \]
\[ P_0 = P \left[ (n_\parallel^2 - S)^2 - D^2 \right] \]

In the electrostatic limit, this reduces to:

\[ k_\perp^2 \approx -\frac{P}{S} k_\parallel^2 \Rightarrow k_\perp \approx \frac{\omega_{pe}}{\omega} k_\parallel \]

In the electrostatic limit, this reduces to:

\[ k_\perp \approx \frac{\omega_{pe}}{\omega} k_\parallel \]

Let’s plug in some numbers:

**Alcator C-Mod typical parameters**

\[ B_0 = 4.5T, D^+ \]
\[ f_0 = 4.6\text{GHz} \]
\[ n_\parallel = 2.5, \; n_a = \frac{\omega_{pe}}{\Omega_{ce}} + \sqrt{S} = 2 \]
\[ n_{e0} = 2 \times 10^{20} \text{m}^{-3} \]
\[ \frac{\omega}{\Omega_D} \approx 125 \]
\[ k_\perp \approx 66\text{cm}^{-1} \]

where the parallel wavenumber, \( n_\parallel \) must be greater than the accessibility criterion, \( n_a \), for the wave to penetrate into the plasma.
LH has severe resolution requirements

- **TORIC** [Brambilla, 1999] discretization is Fourier in flux surface and Hermite finite elements in radial dimension.

\[
\nabla \times \nabla \times \mathbf{E} = \frac{\omega^2}{c^2} \left\{ \mathbf{E} + \frac{4\pi i}{\omega} \left( \mathbf{J}^P + \mathbf{J}^A \right) \right\}
\]

\[
\mathbf{J}^P = \frac{\varepsilon}{\sigma} [f_0(x, v_\perp, v_\parallel)] \cdot \mathbf{E}
\]

This results in a block tridiagonal stiffness matrix, where each block is \((6N_m)^2\), with \(3N_r\) blocks.

- The discrete requirements are estimated from \( \frac{m}{r} \sim k_\perp \sim m \sim 1000\), with similar radial requirements.

- Increase in computational load over ICRF mode conversion simulations \((N_m = 255, N_r = 240)\) of 256 \((\text{ncpu} \sim N_m^3 N_r) \rightarrow \text{parallel processing required}\).
Converged LH Full Wave simulation has been done.

Full Wave Annulus

Loci of LH reflections can be seen in the full wave field patterns, similar to raytracing field structure. There is a suggestion of resonance cones in the full wave field patterns.

Plasma Parameters:

Deuterium gas, \( n_{\parallel} = 1.5 \), \( f_0 = 4.6 \) GHz, \( B_0 = 5.3 \) T, \( T_e = 3.5 \) keV, \( T_i = 2.0 \) keV, \( I = 1 \) MA, \( n_e(0) = 1.5 \times 10^{20} \text{ m}^{-3} \)

See Ref. [WRIGHT et al., 2005] for more details.
**TORIC** algorithm optimized for LH propagation.

- The previous result retained the ion plasma wave branch of the wave dispersion. This is equivalent to having a term proportional to $n_\perp^6$ in the dispersion relation.
- This mode is strongly evanescent and plays not role but increases simulation difficulty.
- To eliminate it, we algebraically eliminate $E_\psi$ in code and drop $dE_\psi/d\psi$ from eqn for $E_\psi$. This decouples $E_\psi$, leaving only two modes.
- In addition, new boundary conditions have been implemented to launch the slow wave polarization:

$$
\frac{c}{\omega} \left[ (\nabla \times E)^{(m)} \right]_{\psi_A} = \frac{4\pi i}{c} J^S \left[ - \sin \Theta_w, \cos \Theta_w \right]
$$

$$
\left[ E^{(m)} \right]_{\psi_A} = - \exp(-im\theta_0) \frac{\exp(-im\Delta_0) + 1}{4m^2\Delta^2_g - \pi^2} \left[ - \sin \Theta_w, \cos \Theta_w \right]
$$
Serial results of new algorithm

- Has much better convergence properties w/o ion plasma wave, but require completion of parallization of algorithm to test.
- Example of launch of slow wave in Alcator-C

Plasma Parameters:

- Hydrogen gas, $n_\| = 1.5$
- $f_0 = 4.6 \text{ GHz}$, $B_0 = 10$
- $T_e = 1.8 \text{ keV}$, $T_i = 1.0$
- keV, $I = 170 \text{ kA}$, $n_e(0) = 0.5 \times 10^{20} \text{ m}^{-3}$
State of impedance calculation

- Using **TORIC** and **TOPICA** codes [ TOPICA (TOrino Polytechnic Ion Cyclotron Antenna) code is a numerical suite aimed at the performance prediction and analysis of plasma-facing antennas. It is capable of handling real-life 3D antenna geometries. See Lacellotti QP1.00047 in this session ] we can find the impedance weighted wave solution as well as the surface fields on the antenna.

- **TORIC** is run with a single \((m, n)\) excitation of a component of \(E_\eta, E_\zeta\) at the plasma surface and the reactive magnetic field components are measured.

- The admittance, \(\overrightarrow{Y}\), is defined as \(\mathbf{B} = \overrightarrow{Y}\mathbf{E}\) for the surface components of \(\mathbf{B}\) and \(\mathbf{E}\) where,

\[
\overrightarrow{Y} = \begin{pmatrix}
\overrightarrow{Y}_{\eta\eta} & \overrightarrow{Y}_{\eta\zeta} \\
\overrightarrow{Y}_{\zeta\eta} & \overrightarrow{Y}_{\zeta\zeta}
\end{pmatrix}
\]
Modal response of TORIC system.

- Each submatrix in the admittance is $N_m N_n$ elements squared for the total number of poloidal and toroidal modes excited separately for each electric field component.
- Because the numerical plasma model is dependant on $n_\phi$ and independent of the boundary condition, the solution or inverse of the stiffness matrix can be used for each separate boundary condition. That is:

$$\leftrightarrow A x_i = b_i \quad b_i \propto \delta_{m\dot{m}}$$

- Each $b_i$ is a column vector in $\leftrightarrow Y_{ij}$. In fact, it can be shown that $Y_{ij}$ is a linear transformation of $A^{-1}$, and so is a representation of the wave-conductivity model.
Modal response of **TORIC** system...cont.

E\_\eta driven at m=5 at surface

B\_\eta response at surface
Progress on Non-Maxwellian ion and electron dielectrics

- See Phillips, Qin, et al. QP1.00037 in this session for the full story.
- Dielectric functions in the Stix form [STIX, 1992] have been tested.
- Asymptotic analysis of hot ion dielectric by Qin done, algorithm implementation next.
Discussion and next steps
References and further reading

Numerical simulation of ion cyclotron waves in tokamak plasmas.


*The Theory of Plasma Waves.*
American Institute of Physics, New York.

Full-wave Electromagnetic Field Simulations of Lower Hybrid Waves in Tokamaks.